A multi-criteria model to analyze decision making in different farm structures: The case of dairy sheep farming in Greece

Sintori A. Rozakis S. and Tsiboukas K.
Department of Agricultural Economics and Rural Development, Agricultural University of Athens, 75 Iera Odos 118 55 Athens, Greece.
Email: al_sintori@yahoo.gr, rozakis@aua.gr, tsiboukas@aua.gr

Abstract
Optimization models commonly used in agricultural studies assume profit maximization as the only objective of farmers. But the existence of diversified farm structures is, at a certain extent, the result of individual preferences and objectives. In this study we aim at building a mathematical model to study the behaviour of Greek sheep farmers. A non-interactive methodology is used to assess the utility function of farmers, which is then optimized subject to the constraint set. The results of the analysis indicate the multi-attribute form of the utility function and point out the ability of the model to accurately reproduce farmer’s behaviour.

Key words: Sheep farming, multi-criteria programming, multiple goals, non-interactive elicitation

Copyright 2010 by Sintori A., Rozakis S. and Tsiboukas K. All rights reserved. Readers may make verbatim copies of this document for non-commercial purposes by any means, provided that this copyright notice appears on all such copies.
1. Introduction

Traditional, single objective, linear programming models are commonly used to capture livestock farmers’ decision making process (Biswas et al., 1984; Conway & Killen, 1987; Alford et al., 2004; Veysset et al., 2005; Crosson et al., 2006). They allow for a detailed technico-economic representation of the farms and take into account interrelationships and physical linkages between alternative production activities. The common characteristic of most optimization models is the assumption that profit maximization is the only objective of farmers. On the other hand, many studies have underlined the existence of multiple goals in agriculture and have focused on the relationship between individual goals and the development of management styles and strategies (Harman et al., 1972; Cary & Holmes, 1982; Fearwheather & Keating, 1994; Costa & Rehman, 1999; Solano et al., 2001; Vandermersch & Mathijs, 2002; Bergevoet et al., 2004).

The existence of different structures even amongst farms with similar activities and constraints correspond to the above findings and is linked to different management strategies developed according to the objectives and preferences of the farmers. Previous studies indicate, for example, that the goals of farmers differ between large and small farms (Gasson, 1973; Wallace & Moss, 2002). Thus, the role of farmers’ objectives on all on-farm decisions and on the development of farm structure is fundamental. Traditional models ignore the multiplicity of objectives in farmers’ decision making and may therefore be less effective or even misleading (Arriaza & Gómez-Limón, 2003).

In this study we suggest the use of a multi-criteria model to study Greek sheep farmers’ decision making. Sheep farming is the most important livestock activity in Greece, located mainly in less favoured areas of the country. The activity contributes highly in the country’s gross agricultural production value and in regional development, especially in isolated and less favored areas (H.M.R.D.F.1, 2007). The main production orientation of Greek sheep farms is milk production, while meat production accounts for less than 40% of the gross revenue of sheep farms (Hadjigeorgiou, 1999; Zioganas et al., 2001; Kitsopanidis, 2006).

A dual farm structure is present in the Greek sheep farming activity, with large commercial and extensive breeding farms on one hand and small scale, family farms on the other (see also Rancourt et al., 2006; H.M.R.D.F., 2007). Recently, more intensive breeding farms have also appeared especially in lowland areas that use more homegrown feed and less pastureland. To account for this heterogeneity of the sheep farming activity, the analysis is undertaken on

---

1 Hellenic Ministry of Rural Development and Food
three farms with different characteristics. The elicitation of the multi-dimensional utility function is attempted using the non-interactive methodology proposed by Sumpsi et al. (1996) and further extended by Amador et al., (1998). The multi-objective farm-level model built can replace traditional single objective models used in agricultural planning. It should be noted that, in our analysis the appropriate from of the utility function is estimated by assessing the performance of the multi-criteria model in the objective space (as suggested by Amador et al. (1998) as well as in the decision variable space.

In the following section the non-interactive, multi-criteria methodology for the elicitation of the utility function is described. Next, the data used in this analysis and the background-model specification are presented. In the last two sections the results of the analysis and some concluding remarks are included.

2. Methodology

Multi-criteria approaches, mainly goal programming and multi-objective programming are common in agricultural studies (McGregor & Dent, 1993; Piech & Rehman, 1993; Siskos et al., 1994; Berbel & Rodriguez-Ocaña, 1998). In these approaches, the goals incorporated in the model and the weights attached to them are elicited through an interactive process with the farmer (Dyer, 1972; Rehman & Romero; 1993). Although this approach is theoretically sound, interaction with the farmer comes with many difficulties, since farmers often find it difficult to define their goals and articulate them (Patrick & Blake, 1980). It has been noted that farmers feel uncomfortable when asked about their goals and are also influenced by the presence of the researcher, which make the self reporting of goals a less suitable approach.

In this study, a well-known non-interactive methodology to elicit the utility function of each farmer is applied (Sumpsi et al., 1996). The methodology is based on the determination of the objectives and their relative importance according to the farmer’s actual and observed behavior. Assume that:

\( x \) = vector of decision variables  
\( F \) = feasible set  
\( f_i(x) \) = mathematical expression of the \( i \)-th objective  
\( w_i \) = weight measuring relative importance attached to the \( i \)-th objective  
\( f_i^{*} \) = ideal or anchor value achieved by the \( i \)-th objective  
\( f_i^{-} \) = anti-ideal or nadir value achieved by the \( i \)-th objective  
\( f_i \) = observed value achieved by the \( i \)-th objective  
\( f_{ij} \) = value achieved by the \( i \)-th objective when the \( j \)-th objective is optimized
\( n_i \) = negative deviation (underachievement of the \( i \)-th objective with respect to a given target)

\( p_i \) = positive deviation (overachievement of the \( i \)-th objective with respect to a given target)

\( D \) = largest deviation of the \( i \)-th objective with respect to a given target

First a set of tentative objectives \( f_{i(x)}, \ldots, f_{q(x)} \) is defined, either through preliminary interviews of farmers or according to the related literature. Then the pay-off matrix is obtained, by optimizing each objective separately, over the feasible set and calculating the value of the other objectives at the optimal solution (Sumpsi et al., 1996). Thus, the first entry of the pay-off matrix is obtained by:

\[
\max f_i(x), \text{ subject to } x \in F
\]

since \( f_1^* = f_{11} \). The other entries of the first column of the matrix are obtained by substituting the optimum vector of the decision variables in the rest \( q-1 \) objectives. In general, the entry \( f_{ij} \) will be acquired by maximizing \( f_j(x) \) subject to \( x \in F \) and substituting the corresponding optimum vector \( x^* \) in the objective function \( f_j(x) \).

The elements of the pay off matrix and the observed values of the objectives are used to build the following system of \( q \) equations. This system of equations is used to determine the weights attached to each objective:

\[
\sum_{j=1}^{q} w_j f_{ij} = f_i \quad i = 1, 2, \ldots, q \tag{2}
\]

\[
\sum_{j=1}^{q} w_j = 1
\]

The solution of this system of equations represents the set of weights to be attached to the objectives so that the actual behavior of the farmer can be reproduced \((f_1^*, f_2^*, \ldots, f_q^*)\). It is common that this system produces no non-negative solution and thus the set of weights has to be alternatively approximated. For this reason, three criteria have been used. The first is the \( L_1 \) criterion according to which the sum of positive and negative deviational variables is minimized (Sumpsi et al., 1996; Amador et al., 1998). The weighted goal programming technique can be used to solve this problem (Appa & Smith, 1973; Sumpsi et al, 1996), as shown below:
The $L_1$ criterion corresponds to the separable and additive utility function (Sumpsi et al., 1996):

$$u_i = \sum_{i=1}^{q} w_i \frac{f_i(x)}{k_i}$$  \hspace{1cm} (4)

$k_i$ is a normalizing factor (for example: $k_i = f_i^+ - f_i^-$), used when the objectives are measured in different units (Rehman & Romero, 1993; Sumpsi et al., 1996; Tamiz et al., 1998).

According to the second criterion, the $L_{\infty}$ criterion, instead of the sum of positive and negative deviational variables, the largest deviation $D$ is minimized (Appa & Smith, 1973). The $L_{\infty}$ criterion corresponds to a Tchebycheff utility function that implies a complementary relationship between objectives (Amador et al., 1998):

$$u_\infty = - Max \left\{ \frac{w_i}{k_i} [f_i^+ - f_i(x)] \right\}$$  \hspace{1cm} (5)

In terms of linear programming the following problem is formed and solved to approximate the weights of the objectives (Appa & Smith, 1973):

$$\text{Min} D \text{ subject to:}$$

$$\sum_{j=1}^{q} w_j f_{ij} + f_i D \geq f_i$$

$$- \sum_{j=1}^{q} w_j f_{ij} + f_i D \geq - f_i$$

$$\sum_{j=1}^{q} w_j = 1$$  \hspace{1cm} (6)

The third criterion used to approximate the weights of the objectives is in fact a compromise between $L_1$ and $L_{\infty}$ ($L_{comp}$) and it is represented by the following linear programming problem (Amador et al., 1998):

$$\text{Min} D + \lambda \sum_{i=1}^{q} \left( \frac{n_i + P_i}{f_i} \right) \text{ subject to:}$$
\[
\sum_{j=1}^{q} w_{ij} + n_{i} - p_{i} = f_{i} \quad i = 1, 2, ..., q
\]  
\[
\sum_{j=1}^{q} w_{ij} f_{ij} + f_{i} D \geq f_{i}
\]
\[
- \sum_{j=1}^{q} w_{ij} f_{ij} + f_{i} D \geq - f_{i}
\]
\[
\sum_{j=1}^{q} w_{j} = 1
\]

The weights obtained by solving this problem are used to derive the utility function which has the following form:

\[
\sum w_{ij} = \begin{bmatrix}
\sum w_{ij} f_{ij} + f_{i} D \\
- \sum w_{ij} f_{ij} + f_{i} D \\
\sum w_{j}
\end{bmatrix} = \begin{bmatrix}
\sum w_{ij} f_{ij} + f_{i} D \\
- \sum w_{ij} f_{ij} + f_{i} D \\
\sum w_{j}
\end{bmatrix}
\]

\[
u_{\text{comp}} = \begin{bmatrix}
\sum w_{ij} f_{ij} + f_{i} D \\
- \sum w_{ij} f_{ij} + f_{i} D \\
\sum w_{j}
\end{bmatrix}
\]

As can be seen from this expression, according to the value of the parameter \( \lambda \) different utility functions are obtained. Specifically, for \( \lambda = 0 \) the utility function becomes a Tchebycheff function, while for a very large number of \( \lambda \) the utility function obtained will be very close to the separable and additive one. For small values of \( \lambda \) the utility function is an augmented Tchebycheff function, since the second term gives a slope to the Tchebycheff function. This way, a well balanced solution is obtained (Amador et al., 1998).

The next step is to validate the model that is to check whether the utility function estimated can accurately reproduce farmers’ behavior. In the case of the separable and additive utility function (equation 4) this is done by maximizing it subject to the constraint set. The results of the maximization are compared to the actual values of the \( q \) objectives. Namely, the following mathematical programming problem is formulated and solved (Sumps, et al., 1996):

\[
\max \sum_{i=1}^{q} \frac{w_{i} f_{i}(x)}{k_{i}} \text{ subject to:}
\]
\[
f_{i}(x) + n_{i} - p_{i} = f_{i} \quad i = 1, 2, ..., q
\]
\[
x \in F
\]

In the case of the Tchebycheff function, the utility function is not smooth and the maximization is performed by solving the next problem (Amador et al., 1998):

\[
\min D \text{ subject to:}
\]
\[
\frac{w_{i} f_{i}^{*} - f_{i}(x)}{k_{i}} \leq D \quad i = 1, 2, ..., q
\]
\[
x \in F
\]
The augmented utility function is also not smooth and the next problem is solved instead of the maximization (Amador et al., 1998).

\[
\text{Min } D + \lambda \sum_{i=1}^{q} \left( \frac{p_i + P_i}{f_i} \right) \text{ subject to } \frac{w_i}{k_i} \left[ f_i^* - f_i(x) \right] \leq D \quad i = 1, 2, ..., q \quad (11)
\]

\[x \in F\]

The results obtained are compared with the actual behaviour of the farmer, not only as far the value of objectives is concerned but also in the decision variable space. If one of the preference functions gives results close to the actual values, then this function is considered the utility function that is consistent with the preferences of the farmer. It should be noted that if the above utility functions cannot reproduce farmer’s behavior, other forms of the utility function should be examined (Sumpsi et al., 1996; Amador et al., 1998).

3. Case study

The analysis requires detailed farm level data, so that all the parameters of the mathematical programming model can be estimated. In this analysis, data comes from three sheep farms located in the Prefecture of Etoloakarnania in Western Greece. Sheep farming is a well established activity in this area, where over 6% of the total sheep milk and lamp meat in Greece is produced and almost 9% of the total number of Greek sheep farms is located (N.S.S.G2., 2000; 2006). The majority of farms in the area have a small flock size, which indicates that sheep farming is often a part time or side activity. Specifically 42% of the farms have a flock size of less than 50 sheep, while less than 9% of the farms have a flock size larger than 200 sheep.

The farms used in this analysis, have been chosen to represent diversified farm structures in terms of size, production orientation and breeding system. Size is determined by the size of the flock and the total cultivated land, the orientation is determined mainly by the contribution of each activity to the total gross margin, while the breeding system is identified according to the amount of forage and concentrates used for animal feeding, amount of on produced feed, pastureland used and labor requirements. Choosing farms with different structures can help identify possible differences in goals and behavior of farmers that follow different management strategies.

---

2 National Statistical Service of Greece
For the above reasons, the first selected farm is commercial and has a large flock size (262 productive ewes). It produces part of the forage (alfalfa) and concentrates (maize) it uses and has an annual milk yield of 135 kg/ewe. The farm is considered semi-intensive, since less than 50% of the feed requirements are met through grazing. The gross margin of the farm is generated mainly by the sheep farming activity. The second farm has a middle size flock (80 ewes) and a lower milk yield but it produces alfalfa and maize not only to cover the needs of the livestock activity but also for sale. This is because the farm is located in a lowland and fertile area and has a high crop yield. Although this farm is a commercial farm, and the owner is a full-time farmer, it has a different production orientation than the large farm, since it aims at the production of feedstock and not only in the production of milk. The breeding system is also different since the farm has limited pastureland and the feed requirements of the flock are met mainly from on produced feed.

Finally, the third farm is a small scale farm, representing only a part-time activity for the owner. In the case of sheep farming in Greece, where 63% of the farms have a small number of livestock, it is necessary to study these farms along with the larger farms and stress any differences between them. The part-time farmer produces no feed and aims only at a supplementary income from sheep farming. Nevertheless, it has a satisfactory milk yield (120kg/ewe) and therefore the gross margin of the activity is quite high. The breeding system resembles the large farm, since the farm uses forage and concentrates, but also pastureland to cover the needs of the flock. In the case of the small farm the feed used is purchased and not on produced. It should be mentioned that the gathered data from the three sheep farms refers to the year 2004-2005 (annual data).

3.1. Model specification

The decision variables and the constraints of the model cover all the livestock and crop activities of the farm, and therefore the whole farm model reflects all the interrelationships between them. Specifically, there are three sets of decision variables included in the model. The first set involves the production of fodder and concentrates (alfalfa and maize), the use of pastureland (area of different kinds of pastureland engaged by the farm) and the monthly consumption of the produced or the purchased forage and concentrates. The second set involves monthly family and hired labor engaged in crop and animal activities, while the last set of decision variables involves the livestock activities of the farm and the area engaged in the production of crops for sale (not consumption in the farm). It should be noted that there are four animal activities incorporated in the model, defined by whether the lambs are sold
after weaning or three months after lambing (rearing) and by whether the ewes are premium eligible or not (see Appendix A).

The main component of the constraint matrix ensures the balance of monthly nutrient requirements (dry matter, NEL$^3$, digestible nitrogen) with the monthly distribution of produced and purchased fodder and concentrates. For the estimation of the nutrient requirements of the flock the methodology described by Zervas et al. (2000) has been used. The second component ensures the availability of the required labor of all livestock and crop activities. Land and policy constraints are also included in the model (total own land, irrigated land, available pastureland, number of premium eligible ewes e.t.c.). It should be mentioned that other livestock linear programming models include similar decision variables and constraints (Conway & Killen, 1987; Alford et al., 2004; Crosson et al., 2006). Since variables that refer to number of animals are constrained to receive only integer numbers, the model used is in fact a Mixed Integer Programming Model. Mixed Integer Programming Models are commonly used, when livestock, crop-livestock and aquaculture farms are studied (Engle, 1987; Shaftel & Wilson, 1990). The mathematical expression of the constraint matrix and the decision variables are presented in Appendix A.

3.2. Initial set of objectives

Six objectives have been used in this analysis, which were determined after preliminary interviews with the farmers. The first one is the maximization of the farm’s total gross margin. Similar objectives have been used in most decision making models (Piech & Rehman, 1993; Berbel & Rodriguez-Ocaña, 1998; Wallace and Moss, 2002; Gómez-Limón et al., 2003). The preliminary interviews also indicate that Greek farmers often place more value on keeping their expenses (mainly variable cost) low, than on making maximum profit. This goal has also been included in our analysis since it has been identified and studied in the past (Piech & Rehman, 1993). The third goal involves the minimization of family labour and it is linked to the increase of farmers’ leisure time. The importance of this goal is stressed in a number of studies of farmers’ goals (Barnett et al., 1992; Wallace, 1998; Gómez-Limón at al., 2003).

The fourth objective is linked mainly with the increasing concern about the quality and hygiene of forage and other concentrates. Farmers, especially those that consume part of their products, or aim to produce and promote quality products; prefer to feed their livestock with forage and concentrates produced on the farm, therefore, the fourth objective is the

$^3$ Net Energy of Lactation (Mj)
minimization of purchased feed. The fifth goal is the minimization of the cost of foreign labour (Piech & Rehman, 1993; Berbel & Rodriguez-Ocaña, 1998). This is a major concern of larger farms that attempt to utilise family labour to increase farm income. Also, hired labour is not always abundant and farmers may need to restrict the size of the flock so as to depend only on family labour. Finally, the sixth objective is the minimization of risk. The role of risk in farm decision making has been stressed in a number of studies and has been identified as one of the most important objectives in farming (Amador et al., 1998; Berbel & Rodriguez-Ocaña, 1998; Gómez-Limón at al., 2003). We adopted the MOTAD function\(^4\) to represent risk in the objective function in order to maintain the linearity in the model. The six goals used in this analysis and their mathematical expressions are given below (also see Appendix A for the indices, parameters and decision variables used):

1. Maximization of gross margin (measured in euros)
   
   \[
   f_1(x) = \text{Max} \sum_{i,t} \text{gr}_{i,t} \cdot \text{mara}_{i,t,sales} + \sum_{r,a} \sum_{i} \text{gr}_{a,r} \cdot \text{anim}_{a,r},
   \]
   
   \[
   f_0(x) = \text{Max} \left( \sum_{g} \text{gr}_{g} \cdot \text{gland}_{g} + \sum_{i,t} \text{rqwc}_{i,t} \cdot \text{feed}_{i,t} + \sum_{l,t} \text{rqwc}_{l,t} \cdot \text{feed}_{l,t} \right)
   \]
   
   \[
   + \sum_{a} \sum_{i,t} \text{rqwc}_{a,t} \cdot \text{anim}_{a,t} + \sum_{i} \sum_{l,t} \text{rqwc}_{i,t} \cdot \text{crop}_{i,t,\text{con,sales}} + \sum_{l} \sum_{l,t} \text{lab}_{l,hire,t} \cdot w_{l,hire}
   \]

2. Minimization of the variable cost (measured in euros)
   
   \[
   f_2(x) = \text{Min} \sum_{g} \text{rqwc}_{g} \cdot \text{gland}_{g} + \sum_{i,t} \text{rqwc}_{i,t} \cdot \text{feed}_{i,t} + \sum_{l,t} \text{rqwc}_{l,t} \cdot \text{feed}_{l,t}
   \]

3. Minimization of the family labour (measured in hours)
   
   \[
   f_3(x) = \text{Min} \sum_{t} \sum_{l} \text{lab}_{l,own,t}
   \]

4. Minimization of the amount purchased forage and concentrates\(^5\)
   
   \[
   f_4(x) = \text{Min} \sum_{f,t} \sum_{l} \text{y}_{f,\text{energy}} \cdot \text{feed}_{f,t}
   \]

5. Minimization of hired labor (measured in hours)

\(^4\)The MOTAD model is based on the “Minimization Of The Absolute values of the negative total gross margin Deviations” from the sample mean (Hazell, 1971). It has been used in the study of Amador et al., (1998) and numerous other risk analyses in agriculture, to approximate risk minimization as an objective of the farmers. The minimization is performed subject to the following constraints and for a given level of gross margin:

\[
\sum_{k=1}^{p} \left( c_{h,k} - g_k \right) x_h + y_{h} \geq 0 \quad \forall \ h
\]

where: \( h = \text{years (sample)} \), \( k = \text{activities for sale} \), \( g = \text{sample mean gross margin for the activities} \).

\(^5\)The variable \( \text{feed}_{i,t} \) refers to kilograms of purchased fodder and concentrates of various types, with different nutritional and energy value. Therefore minimising the sum of all purchased fodder and concentrates would lead to the substitution of low nutritional value crops (used in larger amount) with high nutritional value crops (used in smaller amount). To avoid this mistake we use the parameter \( \text{y}_{f,\text{energy}} \) as a normalizing factor. This means that the 4\(^{th}\) goal expresses the “purchased energy” measured in Mj.
\[ f_5(x) = \text{Min} \sum_j \sum_l \text{lab}_l \cdot \text{hire}_l \]  
(10)

6. Minimization of risk - MOTAD (Hazell, 1971)

\[ f_6(x) = \text{Min} \sum_{h=1}^k y_h \]

4. RESULTS OF THE ANALYSIS

As described in the methodology section, the pay-off matrix is obtained by optimizing each objective separately over the feasible set. The entries of the pay-off matrix together with the observed values of the objectives are then used to build the system of \( q \) equations (2) that will provide the weight of each objective in the utility function. For the large farm, the pay-off matrix is presented in (Table 1). The pay-off matrices of the other two farms are formed accordingly. The three criteria \( (L_1, L_* \text{ and } L_{\text{comp}}) \) are then used to approximate the weights.

As noticed in section 2, the weights obtained, using the \( L_{\text{comp}} \), depend on the value of \( \lambda \). The different utility functions obtained for the various levels of \( \lambda \) in the case of the large farm are indicated in Table 2. Table 2 also contains the weights that derive from \( L_1 \) and \( L_* \) criteria. The weights of the other two farms are estimated accordingly.

These weights are then used to form the utility functions of the three farmers. For each farmer the separable and additive utility function \( (u_1) \), the Tchebycheff function \( (u_*) \) and the augmented Tchebycheff function \( (u_{\text{comp}}) \) are estimated. For the large farm the three forms of the utility function are indicated below:

1. Separable and additive utility function:

\[ u_{1,f} = \frac{0.41}{11,983} \cdot f_1(x) - \frac{0.59}{107,179} \cdot f_2(x) \]

2. Tchebycheff utility function

\[ u_{*,f} = \text{Max} \left\{ \frac{0.10}{11,983} \cdot (41,572 - f_1(x)), \frac{0.13}{4,843} \cdot f_1(x), \frac{0.06}{1,354,251} \cdot f_2(x), \frac{0.71}{7,966} \cdot (f_2(x) - 907) \right\} \]

3. Augmented Tchebycheff utility function

\[ u_{\text{comp},f} = \text{Max} \left\{ \frac{0.07}{11,983} \cdot (41,572 - f_1(x)), \frac{0.14}{4,843} \cdot f_1(x), \frac{0.02}{1,354,251} \cdot f_2(x), \frac{0.77}{7,966} \cdot (f_2(x) - 907) \right\} - 0.13 \left\{ \frac{0.07}{11,983} \cdot f_1(x), \frac{0.14}{4,843} \cdot f_1(x), \frac{0.02}{1,354,251} \cdot f_2(x), \frac{0.77}{7,966} \cdot f_2(x) \right\} \]

The estimated weights reveal that gross margin maximization is a significant attribute in the utility function of the large farm only in the case of the separable and additive utility function. Furthermore, the \( L_1 \) criterion places the highest weight in the minimization of variable cost.
The other two forms of the utility function reveal that the farmer aims mainly at the minimization of risk rather than the maximization of gross margin. This result is coherent with previous studies that emphasize the importance of risk management in agriculture. The minimization of family labour and the minimization of purchased feed receive also a smaller but non negligible weight. The fact that the $L_i$ criterion places the highest weight on the minimization of the variable cost, while the $L_\infty$ and the $L_{comp}$ criteria place the highest weight on the minimization of risk can be explained by the fact that the two objectives lead to similar optimum vectors. This means that whether the farmer aims at minimizing variable cost or minimizing risk his behaviour is similar.

In the case of the middle farm which is also commercial but produces crops as well as livestock products the estimated utility functions that derived from the three criteria are presented below:

1. Separable and additive utility function:

$$u_{i,m} = \frac{0.51}{4,799} f_1(x) - \frac{0.03}{3,643} f_2(x) - \frac{0.46}{4,539} f_4(x)$$

2. Tchebycheff utility function

$$u_{i,m} = - \max \left[ \frac{0.62}{4,799} (21,438 - f_1(x)), \frac{0.38}{4,539} f_4(x) \right]$$

3. Augmented Tchebycheff utility function

$$u_{i,m} = - \left[ \max \left\{ \frac{0.54}{4,799} (21,438 - f_1(x)), \frac{0.46}{4,539} f_4(x) \right\} - 0.45 \left( \frac{0.54}{4,799} f_1(x) - \frac{0.46}{4,539} f_4(x) \right) \right]$$

In the case of the middle farm, the three criteria yield more similar results. In all three forms of the utility function the main attribute is the maximization of gross margin. The minimization of purchased feed also receives a high weight. The additive form of the utility function places a small weight in the minimization of variable cost as well.

Finally, the results of the three criteria for the small farm are presented below:

1. Separable and additive utility function:

$$u_{s,m} = \frac{0.24}{1,850} f_1(x) - \frac{0.76}{315} f_6(x)$$

2. Tchebycheff utility function

$$u_{s,m} = - \max \left[ \frac{0.23}{1,850} (4,494 - f_1(x)), \frac{0.02}{96,502} f_4(x), \frac{0.75}{315} (f_6(x) - 187) \right]$$

3. Augmented Tchebycheff utility function
In the case of the small farm the three criteria yield similar results. The main concern of the farmer is the minimization of risk. This objective receives the highest weight in all three forms of the utility function. The maximization of gross margin receives also a high weight in the three alternative forms of the utility function. The Tchebycheff and the augmented Tchebycheff function place a small weight in the minimization of purchased feed. The results of the small farm are similar to the results of the large farm. The two farms differ in size but are similar as far as the production orientation and the breeding system is concerned. The milk yield is high and therefore farmers aim at livestock production. Also the feed requirements are satisfied partly from grazing. These two farms place more value on risk management rather than gross margin. The middle farm differs in the production orientation since it produces crops for sale. Crops have a high yield but also high risk. These results indicate that production orientation and specific farm practices, rather than farm size, are linked to different objectives. Also the analysis indicates that sheep farming is an appropriate activity for farmers who are risk averse.

In order to identify the appropriate form of the utility function, for each farmer all three utility functions are optimized subject to the constraint set. The predicted values of all objectives, according to the traditional, profit maximization model and according to the mutli-criteria model, are compared (Amador et al, 1998). But in order to decide on the ability of the multicriteria model to reproduce farmers’ behavior, the decision variable space has to be taken into account as well.

The results of the three utility functions and the gross margin maximizing utility function as well as the actual values of the objectives and the decision variables for the large farm are summarized in Table 3. The last two rows of the table indicate the ability of the model to reproduce the actual behaviour of the farmer. The sum of the absolute deviations of the predicted values from the observed values are first estimated and then the ratio of the deviations (total deviation in the case of the multicriteria model/total deviation in the case of the traditional model) is used to identify whether the performance of the mathematical model has improved through the use of the estimated utility functions (André & Riesgo, 2007). The utility function that yields better results (smallest total deviation) is assumed to be the utility function of the farmer.
In the objective space, the estimated utility functions (all forms) yield better results in all cases compared to the single-objective utility function, since the relative fit index is smaller than one. This means that the multicriteria model can represent the behavior of farmers more accurately than the traditional gross margin maximization model. Specifically, in the case of the large farm the three estimated utility functions \( (u_{i,j}, u_{x,j}, u_{comp,j}) \) yield better results than the traditional model (Table 3). The smallest relative fit index corresponds to the augmented Tchebycheff function \( (u_{comp,j}) \) which is accepted as the farmer’s utility function. The variable space verifies this result, since the other two estimated functions fail to reproduce the behaviour of the farmer. But the augmented Tchebycheff function has a relative fit index smaller than one in the variable space as well.

Table 4 summarizes the results for the middle farm. In the case of the middle farm, all three estimated utility functions have an increased ability to reproduce the behaviour of the farmer, compared to the traditional model. The relative fit index is smaller than one not only in the objective space but also in the case of the variable space. But, in the case of the middle farm the separable and additive form of the utility function yields better results. This means that \( u_{i,m} \) is considered the utility function of the specific farmer.

Finally, in the case of the small farm the analysis indicates that the utility function of the farmer is the separable and additive one. Specifically, in the case of the objective space the relative fit index of the three utility functions is small indicating that the mathematical model can improve through the use of the utility functions. But, as far as the variable space is concerned the performance of the model is improved only in the case of the \( u_{i,s} \), which is considered the utility function of this farmer. The results of the analysis for the small farm are summarized in Table 5.

It should be mentioned that the analysis reveals a clear link between the sheep farming activity and the risk aversion of the farmers. Indeed livestock products face fewer price fluctuations in Greece, and farmers that are risk averse prefer this activity to crop production. Furthermore, the analysis indicates that the separable and additive form of the utility function is a good approximation of the utility function of the sheep farmers, since for two out of three farmers this is the form of the utility function that yields better results.

Finally, it should be mentioned that the specific farm structure of the sheep farms are better approximated through the use of the multicriteria model. This is obvious in all three cases since the traditional, gross margin maximization model, overestimates the size of the farm.
Especially, in the case of the small farm, the traditional model insinuates a very different farm structure, since the size of the flock is almost twice as big as the actual size. This could lead to significant deviations of the predicted behaviour of the farmer from the actual behaviour, if the traditional model was used for example to estimate impact from the implementation of a new farm policy.

5. **Concluding Remarks**

In this study the elicitation of the utility function of sheep farmers’ and the formation of a multicriteria model that can be used to analyze their behavior is attempted. For this reason a detailed, whole-farm model, adapted to livestock was built that incorporates decision variables and constraints for all animal and crop activities. The elicitation of the utility function is undertaken through a non interactive methodology, so that the drawbacks of the interactive methods can be limited. The weights attached to the objectives of the sheep farmers are estimated using the actual values of the objectives and the multi attribute utility function is then used to reproduce their behavior. The analysis was undertaken in three sheep farms that represent different farm structures, so that the heterogeneity of the objectives and the forms of the utility function can be stressed.

The results of the analysis indicate that sheep farmers aim to achieve multiple goals, one among them is the maximization of gross margin. This objective is the most important attribute of the utility function of only one out of the three farms under study. The other two farms aim mainly at risk minimization. Specifically, the farms that aim mainly at livestock production have a risk averse behaviour. Livestock production is linked to lower risk levels since the price fluctuation is very small compared to the price of crops. The farmer that aims at crop production as well as livestock production has a high crop yield which increases his gross margin significantly. The analysis indicates that diversified farm structures, as far as farm practices like production orientation and breeding system, are concerned are linked to different objectives and forms of utility function.

In this analysis we have assumed three different farm structures. But, the study can be extended to include more farm structures that can derive from a multivariate analysis, like cluster analysis. This way, further conclusions on the link between observed farm structures and farmer preferences and objectives can be drawn.

In general, the analysis indicates that the performance of the mathematical model built to reproduce the operation of a crop-livestock farm can improve through the use of multiple
objectives. This is useful in many practical ways, since it can be used in farm management to develop a realistic scenario for the development of the farms but also in agricultural planning and policy, since it can replace the less accurate single objective models.

References


H.M.R.D.F. (2007). Sheep and goat sector development. [Online]. Available at: http://www.minagric.gr/greek/ENHM_FYLADIA_ZWIKHS/%CE%91%CE%B9%CE%B3%CE%BF%CF%80%CF%81%CE%BF%CE%B2%CE%B1%CF%84%CE%BF%CF%84%CE%BF%CF%86%CE%AF%CE%B1%2013_9_2007.pdf (in Greek). Accessed 28/February/2010


Appendix A

Mathematical expression of the constraints and decision variables of the LP model:

Indices:  
- $ti$: cultivated crops ($P = \{\text{maize, alfalfa, other}\}$)  
- $fi$: cultivated fodder and concentrates ($T = \{\text{maize, alfalfa}\}$)  
- $fs$: purchased fodder and concentrates ($N = \{\text{maize, alfalfa}\}$)  
- $a$: animal activities ($A = \{\text{sheep3, sheep-3}\}$)  
- $r$: animal premiums ($C = \{\text{elig, nelig}\}$)  
- $m$: destination of produced fodder and concentrates ($M = \{\text{con, sale}\}$)  
- $l$: destination of labour ($L = \{\text{crops, flock}\}$)  
- $s$: origin of labour ($S = \{\text{own, hire}\}$)  
- $t$: month  
- $g$: type of pastureland ($G = \{\text{rent, own, com}\}$)  
- $u$: nutritional value ($U = \{\text{dry matter, nitrogen, energy}\}$)

Yield $ti$ crop yield (kg)

$y_{g,u,t}$ nutritional value of pastureland per month (kg)

$y_{b,u}$ nutritional value of produced forage and concentrates (kg)

$y_{p,u}$ nutritional value of purchased forage and concentrates (kg)

$n_{a,t,u}$ monthly feed requirements (kg)

$n_{a,t,u}$ annual feed requirements (kg)

$w_{l,s}$ wage (euros/hr)

$rclab_{b,t}$ monthly labour requirements for crops (hr)

$ralab_{b,t}$ monthly labour requirements for animal activities (hr)

$avail_{t}$ available family labour per month (hr)

$own\_land$ available owned land (stremma$^6$)

$rent\_land$ available pastureland for rent (stremma)

$irr\_land$ irrigated land (stremma)

$gr\_mun$ available communal pastureland (stremma)

$land$ total land (stremma)

$num\_elig$ number of premium eligible ewes (number)

$gr\_marc_{c,r}$ gross margin of crops (gross revenue minus variable cost except labour) (€)

$gr\_mara_{a,r}$ gross margin of animal activities (gross revenue minus all variable cost except labour and feed cost) (€)

$rqwcg$ variable cost required for pastureland (euro/stremma)

$rqwc_{d}$ variable cost required for crops (euro/stremma)

$rqwc_{a}$ variable cost required for animal activities (euro/ewe)

$^6$ 1 Stremma = 0,1 Ha
Monthly cost of produced fodder and concentrates (euro/kg)

cost of purchased fodder and concentrates (euro/kg)

Percent of energy covered from concentrates

Decision variables

crop_{f,con} produced fodder and concentrates for consumption (kg)
crop_{f, sales} crops for sale (stremma)
feed_{f,t} monthly purchased fodder and concentrates (kg)
feed_{f,t} consumption of produced fodder and concentrates/month (kg)
lab_{l,s,t} labour per month, destination and origin (hr)
gland_{g} pastureland (stremma)
anim_{a,r} ewe (number)

The mathematical expression of the constrain matrix is the following:

Distribution of produced feed crops:

\[ \text{yield}_{f} \cdot \text{crop}_{f,con} = \sum_{t} \text{feed}_{f,t} \quad \forall f \in F \]

Feed requirements:

\[ \sum_{b} y_{b, g, t, u} \cdot \text{gland}_{g} + \sum_{f} y_{f, t, u} \cdot \text{feed}_{f, t} + \sum_{f} y_{f, t, u} \cdot \text{feed}_{f, t} \geq \sum_{f} \sum_{a} n_{a, t, u} \cdot \text{anim}_{a, r} \quad \forall t \in T, u \in U \]

Minimum annual energy requirements satisfied from concentrates:

\[ y_{f, energy} \cdot \text{yield}_{f} \cdot \text{crop}_{f,con} + \sum_{t} y_{f, energy} \cdot \text{feed}_{f, t} \geq \text{percent} \_\text{energy} \cdot \sum_{f} \sum_{a} n_{a, t, energy} \cdot \text{anim}_{a, r} \]

fs==corn, fi==corn

Labour requirements for crops:

\[ \sum_{n} \text{rclab}_{n, t} \cdot (\text{crop}_{n, sales} + \text{crop}_{f,con}) \leq \sum_{s} \text{lab}_{crop, s, t} \quad \forall t \in T \]

Available family labour:

\[ \text{lab}_{l, own, t} \leq \text{avail}_{l, t} \quad \forall t \in T \]

Labour requirements of the flock:

\[ \sum_{a} \text{ralab}_{a, t} \cdot \text{anim}_{a, r} \leq \sum_{t} \text{lab}_{t, s, t} \quad \forall t \in T \]

Available irrigated land:

\[ \sum_{a} \text{crop}_{n, sales} + \text{crop}_{f,con} \leq \text{irr} \_\text{land} \]

Available own land:

\[ \sum_{n} \text{crop}_{n, sales} + \text{crop}_{f,con} + \text{gland}_{own} \leq \text{land} \]

Communal pasture land

\[ \text{gland}_{mun} \leq \text{graz} \_\text{mun} \]

Available land for rental:

\[ \text{gland}_{rent} \leq \text{rent} \_\text{land} \]

Number of ewe rights:

\[ \sum_{a} \text{anim}_{a, r} \leq \text{num} \_\text{elig} \]

\text{Pastureland, property of the municipality, distributed among livestock farms according to their ewe rights. In exchange, livestock farms pay a small fee to the municipality.}
## Appendix B

### Table 1. Entries of the pay-off matrix of the Large Farm

<table>
<thead>
<tr>
<th></th>
<th>Max gross margin (Euros)</th>
<th>Min variable cost</th>
<th>Min family labour</th>
<th>Min purchased forage</th>
<th>Min cost of hired labour</th>
<th>Min Risk</th>
<th>Observed values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross margin</td>
<td>41,572</td>
<td>29,758</td>
<td>29,589</td>
<td>29,589</td>
<td>29,589</td>
<td>36,986</td>
<td></td>
</tr>
<tr>
<td>Variable cost</td>
<td>60,949</td>
<td>3,710</td>
<td>110,889</td>
<td>10,907</td>
<td>13,211</td>
<td>5,155</td>
<td>31,680</td>
</tr>
<tr>
<td>Family labour</td>
<td>4,843</td>
<td>4,438</td>
<td>0</td>
<td>3,831</td>
<td>4,151</td>
<td>4,264</td>
<td>4,843</td>
</tr>
<tr>
<td>Purchased forage</td>
<td>786,048</td>
<td>0</td>
<td>1,554,251</td>
<td>0</td>
<td>115,349</td>
<td>0</td>
<td>324,844</td>
</tr>
<tr>
<td>Cost of hired labour</td>
<td>19,680</td>
<td>114</td>
<td>45,809</td>
<td>597</td>
<td>0</td>
<td>518</td>
<td>7,958</td>
</tr>
<tr>
<td>Risk</td>
<td>6,085</td>
<td>943</td>
<td>4,843</td>
<td>8,873</td>
<td>6,769</td>
<td>907</td>
<td>1,954</td>
</tr>
</tbody>
</table>

Source: Author estimations

### Table 2. Weights of the objectives for the Large farm

<table>
<thead>
<tr>
<th></th>
<th>( L_1 )</th>
<th>( L_x )</th>
<th>( L_{comp} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \lambda &lt;0.11 )</td>
<td>( 0.11 \leq \lambda &lt;0.52 )</td>
<td>( 0.52 \leq \lambda &lt;0.78 )</td>
</tr>
<tr>
<td>Gross margin (Euros)</td>
<td>41%</td>
<td>10%</td>
<td>10%</td>
</tr>
<tr>
<td>Variable cost (Euros)</td>
<td>59%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Family labour (Hours)</td>
<td>13%</td>
<td>13%</td>
<td>14%</td>
</tr>
<tr>
<td>Purchased forage (Mj)</td>
<td>6%</td>
<td>6%</td>
<td>2%</td>
</tr>
<tr>
<td>Cost of hired labour (Euros)</td>
<td>71%</td>
<td>71%</td>
<td>77%</td>
</tr>
</tbody>
</table>

Source: Author estimations

### Table 3. Predicted and observed values of the objectives and the decision variables for the Large farm

#### Objective space

<table>
<thead>
<tr>
<th></th>
<th>Max gross margin</th>
<th>( u_{1,j} )</th>
<th>( u_{x,j} )</th>
<th>( u_{comp,j} )</th>
<th>Observed values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross margin (Euros)</td>
<td>41,572</td>
<td>36,477</td>
<td>29,589</td>
<td>29,589</td>
<td>36,986</td>
</tr>
<tr>
<td>Variable cost (Euros)</td>
<td>60,949</td>
<td>31,177</td>
<td>47,537</td>
<td>43,329</td>
<td>31,680</td>
</tr>
<tr>
<td>Family labour (Hours)</td>
<td>4,843</td>
<td>4,843</td>
<td>4,628</td>
<td>4,724</td>
<td>4,843</td>
</tr>
<tr>
<td>Purchased forage (Mj)</td>
<td>786,048</td>
<td>324,844</td>
<td>702,719</td>
<td>578,439</td>
<td>324,844</td>
</tr>
<tr>
<td>Cost of hired labour (Euros)</td>
<td>19,680</td>
<td>7,015</td>
<td>10,627</td>
<td>11,069</td>
<td>7,958</td>
</tr>
<tr>
<td>Risk</td>
<td>6,085</td>
<td>7,860</td>
<td>2,255</td>
<td>2,046</td>
<td>1,954</td>
</tr>
<tr>
<td>Total deviation</td>
<td>6.06</td>
<td>3.17</td>
<td>2.40</td>
<td>1.81</td>
<td>1.00</td>
</tr>
<tr>
<td>Relative fit</td>
<td>1.00</td>
<td>0.52</td>
<td>0.40</td>
<td>0.30</td>
<td></td>
</tr>
</tbody>
</table>

#### Variable space

<table>
<thead>
<tr>
<th></th>
<th>Number of ewes</th>
<th>Alfalfa produced for consumption*</th>
<th>Maize produced for consumption*</th>
<th>Total pasturage*</th>
<th>Crops for sale*</th>
<th>Total deviation</th>
<th>Relative fit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>380</td>
<td>43</td>
<td>9</td>
<td>800</td>
<td>34</td>
<td>7.08</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>213</td>
<td>21</td>
<td>11</td>
<td>800</td>
<td>52</td>
<td>10.87</td>
<td>1.53</td>
</tr>
<tr>
<td></td>
<td>256</td>
<td>16</td>
<td>6</td>
<td>299</td>
<td>59</td>
<td>12.99</td>
<td>1.83</td>
</tr>
<tr>
<td></td>
<td>268</td>
<td>5</td>
<td>24</td>
<td>667</td>
<td>21</td>
<td>4.74</td>
<td>0.67</td>
</tr>
</tbody>
</table>

*Stremmas

Source: Author estimations
Table 4. Predicted and observed values of the objectives and the decision variables for the Middle farm

<table>
<thead>
<tr>
<th>Objective space</th>
<th>Max gross margin</th>
<th>$u_{1,m}$</th>
<th>$u_{w,m}$</th>
<th>$u_{comp,m}$</th>
<th>Observed values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross margin (Euros)</td>
<td>21,438</td>
<td>20,429</td>
<td>20,381</td>
<td>20,274</td>
<td>20,798</td>
</tr>
<tr>
<td>Variable cost (Euros)</td>
<td>7,798</td>
<td>7,505</td>
<td>7,552</td>
<td>7,561</td>
<td>8,153</td>
</tr>
<tr>
<td>Family labour (Hours)</td>
<td>2,756</td>
<td>2,661</td>
<td>2,639</td>
<td>2,639</td>
<td>2,274</td>
</tr>
<tr>
<td>Purchased forage (Mj)</td>
<td>438</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Cost of hired labour (Euros)</td>
<td>438</td>
<td>401</td>
<td>421</td>
<td>423</td>
<td>350</td>
</tr>
<tr>
<td>Total deviation</td>
<td>1.14</td>
<td>0.41</td>
<td>0.80</td>
<td>0.81</td>
<td></td>
</tr>
<tr>
<td>Relative fit</td>
<td>1.00</td>
<td>0.36</td>
<td>0.70</td>
<td>0.71</td>
<td></td>
</tr>
</tbody>
</table>

Variable space

| Number of ewes | 157 | 105 | 105 | 105 | 72 |
| Alfalfa produced for consumption* | 32 | 25 | 25 | 25 | 16 |
| Maize produced for consumption* | 22 | 15 | 15 | 15 | 9 |
| Total pasturage* | 15 | 15 | 15 | 15 | 15 |
| Crops for sale* | 21 | 35 | 35 | 35 | 50 |
| Total deviation | 4.18 | 2.00 | 2.02 | 2.05 | |
| Relative fit | 1.00 | 0.48 | 0.48 | 0.49 | |

Source: Author estimations

Table 5. Predicted and observed values of the objectives and the decision variables for the Small farm

<table>
<thead>
<tr>
<th>Objective space</th>
<th>Max gross margin</th>
<th>$u_{1,s}$</th>
<th>$u_{w,s}$</th>
<th>$u_{comp,s}$</th>
<th>Observed values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross margin (Euros)</td>
<td>4,494</td>
<td>2,728</td>
<td>3,113</td>
<td>3,132</td>
<td>3,263</td>
</tr>
<tr>
<td>Variable cost (Euros)</td>
<td>5,096</td>
<td>2,913</td>
<td>3,401</td>
<td>3,526</td>
<td>3,102</td>
</tr>
<tr>
<td>Family labour (Hours)</td>
<td>952</td>
<td>671</td>
<td>538</td>
<td>533</td>
<td>671</td>
</tr>
<tr>
<td>Purchased forage (Mj)</td>
<td>14,594</td>
<td>73,567</td>
<td>86,082</td>
<td>89,236</td>
<td>73,567</td>
</tr>
<tr>
<td>Cost of hired labour (Euros)</td>
<td>24</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>Risk</td>
<td>502</td>
<td>215</td>
<td>257</td>
<td>262</td>
<td>237</td>
</tr>
<tr>
<td>Total deviation</td>
<td>5.52</td>
<td>1.23</td>
<td>1.51</td>
<td>1.59</td>
<td></td>
</tr>
<tr>
<td>Relative fit</td>
<td>1.00</td>
<td>0.22</td>
<td>0.27</td>
<td>0.29</td>
<td></td>
</tr>
</tbody>
</table>

Variable space

| Number of ewes | 45 | 27 | 31 | 32 | 23 |
| Alfalfa produced for consumption* | 0 | 0 | 0 | 0 | 0 |
| Maize produced for consumption* | 0 | 0 | 0 | 0 | 0 |
| Total pastureland* | 23 | 24 | 25 | 25 | 23 |
| Crops for sale* | 3 | 1 | 1 | 1 | 3 |
| Total deviation | 0.96 | 0.88 | 1.10 | 1.14 | |
| Relative fit | 1.00 | 0.92 | 1.15 | 1.20 | |

Source: Author estimations