## Πίνακας με συνοδευτικές υποστηρικτικές πληροφορίες

<table>
<thead>
<tr>
<th>Τίτλος πράξης</th>
<th>ΘΑΛΗΣ-ΕΚΠΑ-ΑΣΦΑΛΕΙΣ ΑΣΥΡΜΑΤΕΣ ΜΗ-ΓΡΑΜΜΙΚΕΣ ΕΠΙΚΟΙΝΩΝΙΕΣ ΣΤΟ ΦΥΣΙΚΟ ΕΠΙΠΕΔΟ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Κωδικός πράξης</td>
<td>380202</td>
</tr>
<tr>
<td>Τίτλος υποέργου</td>
<td>ΑΣΦΑΛΕΙΣ ΑΣΥΡΜΑΤΕΣ ΜΗ-ΓΡΑΜΜΙΚΕΣ ΕΠΙΚΟΙΝΩΝΙΕΣ ΣΤΟ ΦΥΣΙΚΟ ΕΠΙΠΕΔΟ</td>
</tr>
<tr>
<td>Τίτλος μελέτης (προσδορίστε από μελέτες, εκπαιδευτικό υλικό, εμπειρογνωμοσύνες, αξιολογήσεις κλπ.)</td>
<td>D2.3.3 ΔΗΜΟΣΙΕΥΣΗ: K MODELED TURBULENCE AND NONLINEAR CLIPPING FOR QAM OFDM WITH FSO AND FIBER SERIALLY LINKED</td>
</tr>
<tr>
<td>Αριθμός συνημμένων αρχείων μελέτης</td>
<td>1</td>
</tr>
<tr>
<td>Τίτλοι υποπαραδότεων μελέτης (σε περίπτωση που υπάρχουν)</td>
<td>01/07/2012 - 31/03/2014</td>
</tr>
<tr>
<td>Ημερομηνία εκπόνησης της μελέτης</td>
<td>ΕΘΝΙΚΟ ΚΑΙ ΚΑΠΟΔΙΣΤΡΙΑΚΟ ΠΑΝΕΠΙΣΤΗΜΙΟ ΑΘΗΝΩΝ</td>
</tr>
<tr>
<td>Τελικός δικαιούχος</td>
<td>ΕΘΝΙΚΟ ΚΑΙ ΚΑΠΟΔΙΣΤΡΙΑΚΟ ΠΑΝΕΠΙΣΤΗΜΙΟ ΑΘΗΝΩΝ</td>
</tr>
<tr>
<td>Φορέας υλοποίησης</td>
<td>ΚΑΘΗΓΗΤΗΣ ΝΙΚΟΛΑΟΣ ΚΑΛΟΥΠΤΣΙΔΗΣ</td>
</tr>
<tr>
<td>Ανάδοχος</td>
<td></td>
</tr>
</tbody>
</table>
Deliverable 2.3.3
K Modeled Turbulence and Nonlinear Clipping for QAM OFDM with FSO and Fiber Serial Linked

H.E. Nistazakis, A.N. Stassinakis, G.S. Tombras, S. Sheikh Muhammad, and A.D. Tsigopoulos
K Modeled Turbulence and Nonlinear Clipping for QAM OFDM with FSO and Fiber Serially Linked

H.E. Nistazakis, A.N. Stassinakis, G.S. Tombras
Department of Electronics, Computers, Telecommunications and Control, Faculty of Physics
National and Kapodistrian University of Athens
Athens, 15784, Greece
{enistaz; a-stasinakis; gtombras}@phys.uoa.gr

S. Sheikh Muhammad
National University of Computer and Emerging Sciences (FAST_NU), Lahore, Pakistan
sm.sajid@nu.edu.pk

A.D. Tsigopoulos
Sector of Battle Systems, Naval Operations, Sea Studies, Navigation, Electronics and Telecommunications
Hellenic Naval Academy, Piraeus, 18539, Greece
atsigo@snd.edu.gr (corresponding author)

Abstract—The free space optical (FSO) and more specifically the radio on free space optical (RoFSO) communication systems are becoming very popular because they can achieve high bit rate transmission with low operational and installation cost. The main disadvantage of such systems is their dependence on the atmospheric conditions and more specifically the randomly time varying characteristics of the propagation path through the atmosphere which is, mainly, caused by the turbulence effect and affects significantly the system's availability and performance. On the other hand, the use of optical fiber systems, offer higher bit-rates and security level but their cost is larger. In this work, the performance of a hybrid dual hop optical communication system is investigated. This system consists of a RoFSO communication link which is connected with an optical fiber link part using a regenerator between them. In both links, the modulation technique that is used is the orthogonal frequency division multiplexing (OFDM) with either a 4 or 16 QAM format. The main phenomena that are taken into account are the atmospheric turbulence, which is modelled with K distribution, the nonlinearities of the laser diode which could be modelled by Volterra series and the biasing with the nonlinear clipping at the optical fiber segment. For this system, closed form mathematical expression for the estimation of its BER is derived and numerical results are presented for realistic parameter values.

Index Terms—BER, Clipping Noise, K Distribution, OFDM, Relay, Wireless Optical Communications.

I. INTRODUCTION

The wireless optical communication systems, and more specifically the RoFSO systems, the last few years, are attracting significant research and commercial interest due to their high availability, performance and reliability along with their low installation and operational cost. However, their performance depends strongly on the atmospheric conditions in their operational area and many atmospheric phenomena affect the irradiance of the laser beam which arrives at the receiver, and mitigate the effectiveness of the whole FSO link [1]-[5]. One of them is the atmospheric turbulence and can be characterized as weak, moderate or strong primarily based on the structure parameter of the refractive index fluctuations. Thus, this phenomenon causes the so-called scintillation effect which results in random fluctuations of the received signal intensity and the channel behaves as a fading one, same as the RF wireless systems [1]-[12]. In order to model these fluctuations, many statistical distribution models have been proposed. In this work we consider the K-distribution which is suitable for strong turbulence conditions [13], [14].

On the other hand, the performance of the communication links with optical fibers is not affected by the weather characteristics, but their installation costs are much higher and generally a special licence and significant infrastructure is needed to deploy them.

In this work, we present a hybrid optical OFDM communication system, which consists of a RoFSO link and an optical fiber one, which are connected through a decoding and forward (DF) OFDM relay node [15]. More specifically, we assume that the RoFSO link is used for signal transmission between the transmitter and the DF relay, while the optical fiber link connects the DF relay with the final receiver of the system. The signal, in each subcarrier of the OFDM link is modulated using a QAM scheme. The OFDM DF relay node regenerates the signal and retransmits it, without the noise of the previous part of the system, through the optical fiber. We assume that the main effect which affects the propagation in the RoFSO link is the atmospheric turbulence [12], [16], [17], while for the optical fiber part is the biasing and nonlinear clipping (BAC) process [18], [19]. For this system, we derive closed form mathematical expressions for the estimation of its BER performance metrics.

II. THE CHANNEL MODEL

The whole optical communication system under consideration consists of two segments. The first one is a RoFSO link between the transmitter and the relay node and the second is an optical fiber which connects the relay with the final receiver of the system. Both systems are using QAM OFDM technique and DF relay node [15].
The first part of this communication system is a wireless one, between the transmitter and the DF relay and the propagation path is the atmosphere. Just before the laser transmitter, after the up-conversion to the carrier frequency \( f_c \), the signal for OFDM subcarrier is given as [12], [20]:

\[
s_{OFDM}(t) = \sum_{n=0}^{N-1} s_n(t) = \sum_{n=0}^{N-1} X_n e^{j[\omega_n t + 2\pi f_c t]} = \sum_{n=0}^{N-1} X_n \exp \left[ 2\pi \left( \frac{n}{T_r} + f_c \right) t \right]
\]

for \( 0 \leq t < T_r \)

where, \( \omega_n \) represents each orthogonal frequency, \( T_s \) is the duration of the OFDM symbol and \( X_n = a_n + j\beta_n \) is the symbol of the \( n \)th subcarrier [12]. Depending on the modulation scheme the raw data are mapped and in this work we concentrate on the nonlinearity coefficient of the LD and propagation path is the atmosphere. Just before the laser one, between the transmitter and the DF relay and the one, between the transmitter and the DF relay and the receiver, for each subcarrier of the \( M \)-QAM case with \( M \) equal to 4 or 16. The OFDM information signal modulates the optical signal at the LD transmitter and the transmitted optical power \( P(t) \), is given as, [12]:

\[
P(t) = P \left[ 1 + \sum_{n=0}^{N-1} m_n s_n(t) + a_n \left( \sum_{n=0}^{N-1} m_n s_n(t) \right) \right]^{3/2}
\]

with, \( P_c \) being the average transmitted optical power, \( a_n \) is the nonlinearity coefficient of the LD and \( m_n \) is the optical modulation index (OMI) for each frequency [12], [21]. The optical signal, after its transmission, propagates through the atmosphere and reaches the receiver of the DF relay node and \( n(t) \) is the additive white Gaussian noise (AWGN) and \( I \) is the instantaneous normalized irradiance at the receiver. The value of this quantity does not remain invariant for long time but fluctuates rapidly due to the scintillation effect, caused by the turbulence. Moreover, using (2), the current at the receiver is given as

\[
i(t, I) = I_o \left[ 1 + \sum_{n=0}^{N-1} m_n s_n(t) + a_n \left( \sum_{n=0}^{N-1} m_n s_n(t) \right) \right] + n_{awg}(t)
\]

where \( I_o = \rho \cdot P_c \) is the dc of \( i(t, I) \), \( \rho \) is the PD’s responsivity, \( n_{awg} \) is the AWGN with zero mean and variance \( N_0/2 \), with \( N_0 = 4K_\text{B}T_c/R_i + 2qI + I(RIN) \), [12], and \( K_\text{B} \) is the Boltzmann’s constant, \( T_c \) is the temperature, \( F \) is the receiver’s noise figure, \( R_i \) is the load resistor at the PD, \( q \) is the electron charge and \( RIN \) is the relative intensity noise process [12]. Hence, we can assume that the instantaneous carrier to noise plus distortion which is arriving at the receiver, for each subcarrier of the OFDM, \( CNDR_n \), is given, approximately, as [12]:

\[
CNDR_n(I) = \frac{(m_n \rho L_{awg} P_c I)^2}{2[N_0/T_c]_{awg} + [\sigma_{\text{IMD}}^2]_{awg}}
\]

where the \( [f]_{awg} \) represents the average value. The quantity \( [N_0/T_c]_{awg} \) stands for the optical noise, while the \( [\sigma_{\text{IMD}}^2]_{awg} \) represents the intermodulation distortion which caused by the LD’s nonlinearities and the OFDM’s characteristics. Moreover, the average value \( [CNDR]_{awg} \), is obtained, as, [17]:

\[
[CNDR]_{awg} = \frac{(m_n \rho L_{awg} P_c I)^2}{2[N_0/T_c]_{awg} + [\sigma_{\text{IMD}}^2]_{awg}}
\]

The second part of the optical communication link consists of an optical fiber and connects the transmitter of the DF relay node with the receiver of the total link. The information signal, in this part, propagates using the OFDM technique, as well, and it is bipolar, in general. Thus, this signal could not be used directly in the intensity modulation/direct detection (IM/DD) systems and the addition of a bias is necessary [18], [22], [23]. Hence, the instantaneous envelope of the OFDM waveform, can be assumed to follow a normal distribution with mean \( \mu_N \) and variance, \( \sigma_N^2 \). In order to avoid clipping of negative peaks, a large bias needs to be added and this is a requirement for the optimization of the minimum bias to be added along with suitable nonlinear clipping for performance enhancement. Thus, the analog bipolar waveform, from the digital to analog converter inside the transmitter, is followed by a biasing and nonlinear clipping (BAC) process to make it work as an IM/DD OFDM communication system [18].

For such an OFDM optical communication system which is using an optical fiber, the total BER of an \( M \)-QAM, IM/DD system with BAC process, is given as [18], [24]:

\[
P_{\text{BER}} = \frac{4(1-M^{-1/2})}{\log_2(M)} Q \left( \frac{3\sqrt{M}}{M-1} \right) - \frac{2(1-M^{-1/2})}{\log_2(M)} Q \left( \frac{3\sqrt{M}}{M-1} \right)^2
\]

where \( Q(.) \) stands for the Q-function, \( M \) is the modulation format of the QAM and the effective SNR, \( \gamma_e \), is given as

\[
\gamma_e = \gamma_c + \gamma_{\gamma_c} \left[ (1+\gamma_c)(1+\gamma_e^2) + \gamma_e \right], [18],
\]

including the BAC process, [22], with \( \gamma_c \) being the SNR at the receiver, \( \gamma_c \) the ratio of the transmitting signal, over the nonlinear clipping noise, while \( \gamma_c = V_{\text{dc}}/\sigma_{\gamma_c} \) is the normalized clipping level and \( V_{\text{dc}} \) the biasing voltage [18]. The \( \gamma_c \) is given as, [18]:

\[
\gamma_c = \frac{2e^{-\gamma_e/2}}{\sqrt{2\pi}} Q(\gamma_e) - \frac{e^{-\gamma_e/2}}{\sqrt{2\pi}} \int \gamma_e Q(\gamma_e) - \int \gamma_e Q(\gamma_e)
\]

III. THE K-DISTRIBUTION MODELED ATMOSPHERIC TURBULENCE EFFECT

Thus, the irradiance of the optical signal of the RoFSO link at the PD’s input depends strongly on the atmospheric conditions of the area of the link. One significant phenomenon that affects strongly the link’s performance is the atmospheric turbulence which causes the scintillation effect that is responsible for the signal’s irradiance fluctuations at the receiver. Depending on the strength of the turbulence, these fluctuations can be modeled with many statistical distribution models. In this work, we assume strong turbulence conditions and the K-distribution [13], [14] and its probability density function (PDF) for the normalized irradiance is given as [14]:

\[
f_c(I) = \frac{2a}{F(a)} F^{a-1}(2\sqrt{I})
\]

where \( K_{a,c}(.) \) stands for the BesselK function and the parameter \( a \), depends on turbulence strength and larger values of \( a \) correspond to weaker turbulence [13], [14].
IV. THE BER OF THE TOTAL COMMUNICATION SYSTEM

In order to estimate the BER performance metric for the whole communication system, the expressions for the wireless OFDM link and the optical fiber link must be derived and finally the total expression should be evaluated. For the estimation of the average BER for the RoFSO link, we assume that the noise obtained in (3) and (4) is Gaussian and the BER per OFDM subcarrier, \( P_{\text{BER},\text{OFDM}} \), is obtained from (5). Moreover, using the approximation of [25] for the Q-function, we conclude to the following integral for the estimation of average BER of the OFDM RoFSO link:

\[
\Big[ P_{\text{BER},\text{OFDM}} \Big]_{\text{AV}} = \frac{2(1 - \sqrt{M^{-1}})}{N \log_2(M)} \sum_{I=0}^{N-1} \left\{ 2Q \left( \sqrt{3CNDR_{\text{OFDM}}(I)} \right) - \frac{1}{18} F_{\text{G}}(48) - \frac{1}{2} F_{\text{G}}(64) - \frac{1}{3} F_{\text{G}}(56) \right\}
\]

By substituting (7) into (8), and transforming \( Q(J), \exp(J) \) and BesselK functions to the corresponding Meijer ones, [26], we solve the above integral and we derive the following closed form mathematical expression for the BER estimation of the OFDM RoFSO link:

\[
\Big[ P_{\text{BER},\text{OFDM}} \Big]_{\text{AV}} = \frac{2^{5/4} \sqrt{\frac{1}{3} \left( 1 - \sqrt{M^{-1}} \right)^2}}{\pi N \log_2(M)} \sum_{I=0}^{N-1} \left\{ 4 \Phi \left( \frac{3CNDR_{\text{OFDM}}(I)}{M^{-1}} \right) - \frac{1}{2} F_{\text{G}}(48) - \frac{1}{2} F_{\text{G}}(64) - \frac{1}{3} F_{\text{G}}(56) \right\}
\]

with \( F_{\text{G}}(A) = G_{\text{av}}^4 \left( \frac{ACNR_{\text{OFDM}}(I)}{(M-1)a^2} - \frac{1}{2}, \frac{2-a}{2}, 0, 0.5 \right) \)

\[
\Xi = G_{\text{av}}^2 \left( \frac{24(3CNDR_{\text{OFDM}}(I))}{(M-1)a^2} - \frac{1}{2}, \frac{2-a}{2}, 0, 0.5 \right).
\]

The BER of the whole hybrid OFDM optical communication system is estimated as [15]:

\[
P_b = P_{\text{BER},\text{OFDM}} + P_{\text{BER},\text{OFDR}} - 2P_{\text{BER},\text{OFDR}} \Big[ P_{\text{BER},\text{OFDR}} \Big]_{\text{AV}}
\]

Next, using (5), (9) and (10), and substituting \( \Xi = Q \left( \frac{3\sqrt{3}}{M-1} \right) \) and \( \Psi = 1 - \sqrt{M^{-1}} \), we derive the following closed form mathematical expression for the estimation of the total BER of the dual hop QAM OFDM optical communication system including an RoFSO over strong turbulence channels modeled with the K-distribution, an optical fiber link with nonlinear clipping noise and a DF relay node:

\[
P_{\text{BER},\text{OFDR}} \approx \frac{2^{5/4} \sqrt{\frac{1}{3} \left( 1 - \sqrt{M^{-1}} \right)^2}}{\pi N \log_2(M)} \sum_{I=0}^{N-1} \left\{ 2^{5/4} \left( 1 - \sqrt{M^{-1}} \right)^2 - \frac{1}{2} F_{\text{G}}(48) - \frac{1}{2} F_{\text{G}}(64) - \frac{1}{3} F_{\text{G}}(56) \right\}
\]

V. NUMERICAL RESULTS

In this section, we are using expression (11) and we present the corresponding results for the BER of the hybrid optical link with DF relays over turbulence channels modeled with the K-distribution. We choose three values for the parameter \( \gamma \) of the BAC process, i.e. 6dB, 9dB, 12dB, two values for \( M \), i.e. 4, 16, while the value of the parameter \( a \) takes the values 1.1 and 3.1 [13]. For the whole optical link, the most significant quantities are CNDR and SNR at the receivers. Obviously, each one can vary independently and the BER for the whole link can be estimated for any CNDR and SNR values, through the expression (14), but in this work we present results assuming, equal values for CNDR and SNR at the receivers.

![Figure 1: BER estimation for the 4-QAM OFDM system.](image1)

![Figure 2: BER estimation for the 16-QAM OFDM system.](image2)
corresponding results with those of Fig. (1), for an OFDM, 16QAM system. It is clear that for low SNR and CNDR values, the BAC process at the optical fiber part, plays a very significant role at the system's BER, while, for larger values, the main mitigation factor is the turbulence at the RoFSO.

VI. CONCLUSIONS

In this work we study the performance of a hybrid OFDM optical system with a regenerator between the RoFSO part and the optical fiber. We assumed that the main mitigation effect for the RoFSO is the atmospheric turbulence, modeled with the K-distribution in this case, while, for the optical fiber part was the nonlinear BAC process. We derived closed form mathematical expressions for the evaluation of its average BER, which is a significant metric for the system's performance estimation. Moreover, using this expression, we presented numerical results for many link's parameter values.

ACKNOWLEDGMENT

This research has been co-financed by the European Union (European Social Fund – ESF) and Greek national funds through the Operational Program "Education and Lifelong Learning" of the National Strategic Reference Framework (NSRF) - Research Funding Program: Thales, Investing in knowledge society through the European Social Fund.

REFERENCES