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BER Estimation of Dual Hop QAM OFDM ROFSO Over Exponentially Modeled Turbulence and Optical Fiber with Nonlinear Clipping

A.N. Stassinakis, M.P. Ninos, H.E. Nistazakis, S. Sheikh Muhammad, A. D. Tsigopoulos, and G.S. Tombras
BER ESTIMATION OF DUAL HOP QAM OFDM ROFSO OVER EXPONENTIALLY MODELED TURBULENCE AND OPTICAL FIBER WITH NONLINEAR CLIPPING

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Abstract. The radio on free space optical (RoFSO) communication systems are gaining popularity due to their high data rates, license free spectrum and adequate reliability at installation and operational costs which are much lower than comparable technologies. A significant disadvantage of these systems concerns the randomly time varying characteristics of the propagation path mainly caused by the turbulence effect. On the other hand, the optical fiber communication systems offer links with higher data rates but with expensive infrastructure and installation cost. In this work, we study the BER performance of an optical communication system which consists of a RoFSO link that is connected with an optical fiber link through a regenerator node. The signal propagates, in both links, using the OFDM technique with QAM format and the dominant impairments which have been taken into account are the atmospheric turbulence, modelled with the negative exponential distribution, the nonlinear responsivity of the laser diode which can be modelled with a Volterra series and the biasing with nonlinear clipping noise. For this setup, closed form mathematical expression for the estimation of system's BER has been derived and the corresponding numerical results are presented for common link parameters.

1 INTRODUCTION

The free space optical (FSO) and more specifically the RoFSO communication systems attract significant research and commercial interest the last years due to their high performance capabilities, high security level that they offer, the low operational and - without need of licence-installation cost. On the other hand, their efficiency depends strongly on the atmospheric conditions in the specific area due to the wireless signal propagation. Thus, it is well known that a variety of atmospheric phenomena affect the irradiance of the laser beam at the receiver’s side, and reduce the effectiveness of the FSO link [1]-[2].

A significant phenomenon is the atmospheric turbulence characterized as weak moderate or strong. More specifically, it causes the so-called scintillation effect which results in random fluctuations of the received signal intensity and thus the channel can be characterized as a fading one [1]-[2]. Many statistical distribution models have been proposed in order to model these signals, fluctuations according to turbulence strength. In this work we consider the negative exponential (NE) distribution model which is suitable for saturated atmospheric turbulence conditions [11]-[13].

On the other hand, the performance of the optical communication links which are using the optical fibers does not depend on the weather characteristics and thus, offer higher data rate transmission and longer propagation distances, than the FSO links, but their installation costs is much expensive and the most of the times, a special licence and significant infrastructure is needed.

Taking into account the advantages of the above mentioned optical communication links, i.e. RoFSO and optical fiber links, in this work we present a dual hop link which consists of an RoFSO part, a decoding and forward (DF) relay node and an optical fiber link until the signal arrival at the receiver. Thus, we assume that the RoFSO link is needed for signal transmission between two points, where the optical fiber could not be installed, while the -short- optical fiber link transports the information to the specific place where the final receiver is established. The signal in each part of the system is propagating using an orthogonal frequency division...
modulation (OFDM) scheme, in each subcarrier of which the information signal is modulated using the quadrature amplitude modulation (QAM) technique. The DF relay node receives, recognizes and retransmits the optical signal without the noise which has been added in the previous part of the system. Taking into account the dominant influence of the atmospheric turbulence effect at the efficiency mitigation of the RoFSO links we assume that this phenomenon affects significantly the performance of the optical wireless link \[^{[10]-[14]}\]. On the other hand, for the -relatively short- optical fiber link, the biasing and nonlinear clipping (BAC) process is a very important mitigation factor. Thus, we derive closed form mathematical expressions for the estimation of the bit error rate (BER) of the whole serially relayed optical link, for the NE atmospheric turbulence model and the nonlinear BAC effect \[^{[15]-[16]}\].

2 THE CHANNEL MODEL

The whole optical communication link, consists of an optical transmitter which transmits the signal through a wireless optical channel, using an OFDM scheme, the DF relay node, which re-transmits the information, using OFDM technique again, through a non-linear optical fiber. In the first wireless segment between the transmitter and the DF relay node, the propagation medium is the atmosphere. Each of the \(N\)-subcarriers of the OFDM signal, just before the laser transmitter, after up-conversion to the carrier frequency \(f_c\) is given as \[^{[10]}\]:

\[
s_{OFDM}(t) = \sum_{n=0}^{N-1} s_n(t) = \sum_{n=0}^{N-1} X_n \exp \left[ (\omega_n + 2\pi f_c t) \right] \quad \text{for} \ 0 \leq t < T_s
\]

where, the quantity \(\omega_n = 2\pi n/T_s\), \(n = 0, \ldots, N-1\), determines each orthogonal frequency, \(T_s\) is the duration of the OFDM symbols and \(X_n\) is the complex data symbol of the \(n_{th}\) subcarrier \[^{[10]}\]. In this work an \(M\)-ary QAM modulation is used for every subcarrier and \(M\) equals to 4 or 16. The total optical power \(P(t)\) transmitted from the laser diode LD, follows a nonlinear law and is given as \[^{[10]}\]:

\[
P(t) = P_r \left[ 1 + \sum_{n=0}^{N-1} m_n s_n(t) + a_3 \left( \sum_{n=0}^{N-1} m_n s_n(t) \right)^3 \right]
\]

where, \(P_r\) is the average transmitted optical power, \(a_3\) corresponds to the third order nonlinearity coefficient of the LD and \(m_n\) is the optical modulation index (OMI). The received optical signal in the DF’s relay node receiver after the atmospheric propagation is calculated by the relation \(P_r(t) = P(t) \cdot L_{opt} + n(t)\) \[^{[10]}\], with \(L_{opt}\) being the total losses caused by the atmosphere, \(n(t)\) the additive white Gaussian noise (AWGN) of the channel \[^{[1]}-\[^{[2]}\] \[^{[10]}\] and \(I\) represents the instantaneous normalized irradiance at the receiver which fluctuates rapidly due to the scintillation effect. The output current of the photo detector PD at the receiver of the DF relay node is \[^{[10]}\]:

\[
i(t, I) = I_0 \left[ 1 + \sum_{n=0}^{N-1} m_n s_n(t) + a_3 \left( \sum_{n=0}^{N-1} m_n s_n(t) \right)^3 \right] + n_{opt}(t)
\]

where \(I_0 = \rho L_{opt} P_r \) is the dc of the received photocurrent \(i(t,I)\), \(\rho\) represents the PD’s responsivity, while \(n_{opt}\) is the AWGN with zero mean and variance \(N_0/2\), with \(N_0 = 4K_B T F R_I + 2qL_0 L_c^2 (RIN)\). In this expression, \(K_B\) is the Boltzmann’s constant, \(T\) is the temperature, \(F\) is the noise figure of the receiver, \(R_I\) is the load resistor at the PD’s side, \(q\) is the electron charge and \(RIN\) stand for the relative intensity noise process \[^{[10]}\]. Thus, the received instantaneous carrier to noise plus distortion for each subcarrier of the OFDM, \(CNDR_n\), is given approximately as \[^{[10]}\] \[^{[14]}\]:

\[
CNDR_n(I) \approx \frac{m_n^2 \rho^2 L_{opt}^2 P_r^2 T^2}{2[N_0/T_S]_AV + [\sigma^2_{IMD}]_AV}
\]

where the symbol \(f_{J_{4V}}\) declares the average value and \(\sigma^2_{IMD}\) stands for the inter-modulation distortion (IMD), due to the nonlinear responsivity of the LD, which affects the specific carrier \(\omega_n\) among \(N\) equally spaced subcarriers of the OFDM scheme and depends strongly on the third order nonlinearity coefficient, \(a_3\), and the OMI for each subcarrier, \(m_n\). From (4), the average value \(f_{CNDR_nJ_{4V}}\), can be obtained considering that the average value of \(I\) is normalized to unity \[^{[11]}\]. In this case it is given as:
The second segment of the optical communication link between the DF relay node and the receiver consists of an optical fiber. The information signal is conveyed with the OFDM technique again, which is bipolar in general. In this form the signal is not suitable for IM/DD systems, and a bias is added in order to overcome this problem \([15]-[19]\). So, the instantaneous OFDM waveform is considered to follow a normal distribution with mean \(\mu_N\) and variance \(\sigma_N^2\). The undesired effect of frequent clipping of negative peaks is avoided by adding a large bias. This is a requirement for the optimization of the minimum bias to be added along with suitable nonlinear clipping for increasing performance. Hence, the analog bipolar waveform of the digital to analog converter from the transmitter is followed by a biasing and nonlinear clipping BAC process to make it work as an IM/DD OFDM communication system.

In this part of the optical fiber, the total BER of the OFDM IM/DD system with \(M\)-QAM modulation is given as \([20]\):

\[
P_{s,OF} = 4 \frac{(1 - M^{-1/2})}{\log_2(M)} Q\left(\frac{3\gamma_e}{\sqrt{M-1}}\right) - \left[2 \left(1 - M^{-1/2}\right)\right] Q\left(\frac{3\gamma_e}{\sqrt{M-1}}\right)^2
\]

With \(Q(.)\) being the Q-function, \(M\) is the signal constellation of the QAM and \(\gamma_e\) the effective SNR which is given as \(\gamma_e = \gamma_d \left[\left(1 + \gamma_e\right) \left(1 + \gamma_c^2\right) + \gamma_d\right]\), including the BAC process, \([15][21]\), where \(\gamma_d\) is the SNR at the receiver, \(\gamma_c\) the ratio of the transmitting signal, from the regenerator node, over the nonlinear clipping noise, and \(\gamma_e = V_{DC}/\sigma_N\) stands for the normalized clipping level with \(V_{DC}\) the biasing voltage \([15][21]\). The value of \(\gamma_e\) is given as \([15][21]\):

\[
\gamma_e = \left[\frac{2\gamma e^{-\gamma^2/2}}{\sqrt{2\pi}} Q(y) - \frac{e^{-\gamma^2/2}}{\sqrt{2\pi}} + \left(\gamma^2 + 1\right)Q(y) - \left(\gamma^2 + 1\right)\left[Q(y)\right]^2\right]^{-1} - \left[1 - Q(y)\right]^2
\]

3 THE EXPONENTIALLY MODELED TURBULENCE CHANNEL

As we have mentioned above the atmospheric turbulence can cause significant degradation to the link’s performance. The refractive index variations of the medium also known as scintillation effect cause fluctuations to the signal’s irradiance at the receiver. The amount of normalized irradiance \(I\) quantifies this variations and according to the atmospheric turbulence strength these fluctuations can be characterized as weak, moderate and strong. Many statistical distribution models have been proposed for these irradiance fluctuations. The negative exponential NE is one of them and it is suitable for saturated turbulence conditions \([11]-[13]\). Its probability density function (PDF), for the normalized irradiance \(I\), is given as, \([11]\):

\[
f_I(I) = \exp(-I)
\]

4 THE TOTAL AVERAGE BER

A significant metric for the estimation of the system’s performance is the BER \([22][23]\). For the average BER’s estimation, we assume that the total noise included in \((4), (5)\) is AWGN, for the \(M\)-QAM. Thus, to calculate the average BER of each OFDM subcarrier of the total multihop wireless optical communication system, \([P_{s,FSO}]_{AV}\), by assuming Gray-coded mapping at the transmitter’s side, we are using the expression of \((6)\) and is given as \([14]\):

\[
[P_{s,FSO}]_{AV} = \frac{2 \left[1 - M^{-1/2}\right]}{\sqrt{2\pi} M \log_2(M)} \sum_{n=0}^{N-1} \int \left[\text{erfc}\left(\frac{3\text{CNDR}_n(l)}{\sqrt{2(M-1)}}\right) - \frac{1}{2} \left(1 - \sqrt{M}e^{-l}\right) \text{erfc}\left(\frac{3\text{CNDR}_n(l)}{2(M-1)}\right)\right] f_I(l) dl
\]

where \(M\) is the modulation format value for the \(M\)-QAM. In order to calculate the \(\text{erfc}(.)\) the following approximation will be used \([24]\):

\[
\text{erfc}(x) = \frac{1}{6} e^{-x^2} + \frac{1}{2} e^{-\frac{4}{3}x^2}
\]
\[
\text{erfc}^2(x) = \frac{1}{36} e^{-2x^2} + \frac{1}{4} e^{-\frac{8}{3}x^2} + \frac{1}{12} e^{-\frac{7}{3}x^2}
\] (11)

By transforming expression (11) to the corresponding Meijer function \cite{25}, using the PDF of (8) and solving the integral (9), we conclude to the following form:

\[
[P_{b,FSO}]_{AV} = \frac{2}{N \log_2(M)} \left[ \prod_{n=0}^{N-1} \left\{ \frac{1}{\pi} \sqrt{\frac{\Theta - \sqrt{\frac{2}{\pi}} (1 - \sqrt{M})}{2 \sqrt{L_i}}} \right\} \exp \left( \frac{1}{4L_i} \right) \text{erfc} \left( \frac{1}{2\sqrt{L_i}} \right) \right]
\] (12)

With \( \Theta = G^2 (\frac{6}{M-1} \left[ 0.5, 1 \right], \left( 0, 0.5 \right]) \) and
\( L_1 = \frac{3}{M-1} \left[ \text{CNDR}_a \right]_{AV}, L_2 = \frac{4}{M-1} \left[ \text{CNDR}_a \right]_{AV}, L_3 = \frac{7}{2(M-1)} \left[ \text{CNDR}_a \right]_{AV} \)

The BER estimation of the whole OFDM optical link is given by the use of the following expression \cite{22,26}:

\[
P_b = P_{b,FO} + \left[ P_{b,FSO} \right]_{AV} - 2P_{b,FO} \left[ P_{b,FSO} \right]_{AV}
\] (13)

By substituting (6) and (12) into (13), and replacing with \( \Omega = \sqrt{\frac{3\gamma}{M-1}} \) and \( \Psi = 1 - \sqrt{M^{-1}} \) we obtain the following closed form mathematical expression for the total BER estimation of the hybrid OFDM-QAM system which consists of a RoFSO under saturated turbulence conditions modelled with the Negative Exponential distribution, an optical fiber link affected by the nonlinear clipping noise and a DF relay node:

\[
P_b = \frac{4\Psi}{N \log_2(M)} \sum_{n=0}^{N-1} \left[ 1 - \Psi \Theta \right] \left[ 1 - \frac{8\Psi}{N \log_2(M)} \sum_{n=0}^{N-1} \left[ 1 - \Psi \Theta \right] \right] \times 
\frac{2\Psi}{N \log_2(M)} \sum_{n=0}^{N-1} \left\{ \frac{1}{\pi} \sqrt{\frac{\Theta - \sqrt{\frac{2}{\pi}} (1 - \sqrt{M})}{2 \sqrt{L_i}}} \right\} \exp \left( \frac{1}{4L_i} \right) \text{erfc} \left( \frac{1}{2\sqrt{L_i}} \right)
\] (14)

5 NUMERICAL RESULTS

In this section we present the BER results derived by expression (14) for the dual hop optical communication system. The parameter values introduced are \( \gamma = 6\text{dB}, 9\text{dB} \) and \( 12\text{dB} \) which stands for the strength of the BAC process \cite{15,21}, and two values for the \( M \) parameter \( M = 4, 16 \) which determines the constellation of the M-QAM modulation for each subcarrier. The CNDR and SNR at the receivers are the quantities which we vary in order to determine the BER behavior. The following plots are designed in the range of 0-30 dB for both CNDR and SNR values. Obviously, these quantities can take different values for the corresponding sections that they refer. The assumption that we make in this work is for equal values for CNDR and SNR at the receivers.
In Fig. (1), we present the BER response of the whole optical link for different values of CNDR and SNR, taking into account the three values for $\gamma$ of an OFDM 4QAM, while in Fig. (2) we show the case of an OFDM 16QAM system. As we can observe from the two plots, the $\gamma$ parameter of the BAC process at the optical fiber is a significant mitigation factor for low SNR and CNDR values. For larger values the atmospheric turbulence dominates and determines the system's BER behavior.

6 CONCLUSIONS

In this work we present a hybrid OFDM, dual hop optical communication link, which includes a RoFSO segment, a regenerator DF relay node and an optical fiber part. For the wireless link we assume that the main distortion factor is the atmospheric turbulence, modeled with the Negative Exponential distribution, while for the optical fiber is the nonlinear BAC process. We obtain a closed form mathematical expression for the estimation of the average BER of the whole optical system. From this expression we present some numerical results for different link's parameter values.

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