Euclid’s time of action

This is the School of Athens, the famous fresco by Raphael and  this gentleman here is Euclid of Alexandria, arguably the most famous mathematician of the ancient world, perhaps along with Archimedes. In fact, Euclid is so famous, that, even nowadays, his name is everywhere:  on a space satellite, on street signs, on corporate logos and so on. Just name an object, and Euclid is probably already there! Nevertheless, unfortunately, only a handful of information about the life of this influential man is extant. We have no indication about his place of birth or death, about his physical characteristics, or about his education.

His place of action is inferred by Pappus’ remark that: ‘(Apollonius) spent much time studying with the pupils of Euclid in Alexandria’ and his time of action can be assembled from a series of speculations, most of which are based on a famous account by Proclus: ‘(Euclid) γέγονε in the time of Ptolemy the First’. Surprisingly perhaps, the much later Arabic tradition is more descriptive: al-Qifti, a 12th-century scholar, appears to know details of Euclid’s family background such as the name of his father and grandfather—Naucrates and Zenarchus, respectively. Moreover, according to this account, Euclid was Greek, born in Tyre, and domiciled at Damascus. Al-Nadim, two centuries earlier, records a different name for Euclid’s grandfather: ‘Berenicus’. For many reasons, the Arabic accounts have been approached with scepticism as they are considered to be the result of romanticism and misunderstanding; thus, I propose to leave them aside for the moment, and return to Proclus.

The verb γίγνομαι, means either ‘come into being’ or ‘to be born’; therefore, according to this account, Euclid was either born or flourished during the time of Ptolemy the First. Unfortunately, the time-frame of Ptolemy is also ill-defined: he reigned in Egypt from 306-285 BCE, but he was the ruler of the land already by 323 BCE. Thus, assuming a lifetime of 60 years and a period of flourish around the age of 40, we have four possible scenarios.  I do not wish to engage now the details of the debate about Euclid’s dating; I just want to indicate some of the difficulties a historian has to face when dealing even with the most basic information regarding his life. To make a long story short, it is generally accepted that Euclid lived in Alexandria around 300 BCE, which is about 50 years after the death of Plato.

Proclus’ Euclid

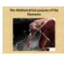
A prevalent idea among contemporary scholars is that Euclid belonged to, or was heavily influenced by the Platonic philosophical tradition. To a certain extent, this idea is not new; the quest for Euclid’s philosophical background was probably triggered and enhanced by his late commentators, Greeks and Arabs, who appear confident that he had one.  For instance, al-Qifti writes: ‘Euclid...called the author of geometry, a philosopher of somewhat ancient date...’ and al-Nadim, in the *Fihrist*, names Euclid as ‘...one of the mathematical philosophers...’ Proclus, five centuries earlier than al-Nadim, was more specific: ‘[Euclid] was a follower of Plato by choice, and familiar with this philosophy’.

It is true that Proclus, a neo-Platonic philosopher and director of the Academy, is not always reliable when ascribing Platonic beliefs to famous people of the past, especially when he appears to have no direct information about Euclid’s personal life. Nevertheless, this time, the details are too rich to be dismissed, and, as Proclus exposes the reason of his belief, this account can be examined *per se*: he deduces that Euclid was a follower of Plato, on the basis of what he sees as the mathematical purpose of the *Elements*; namely, the construction of the so-called Platonic figures. Let me briefly explain this point. The last book of the *Elements*, book XIII, is devoted to the properties of the five regular polyhedra; namely, the tetrahedron, the cube, the octahedron, the icosahedron, and the dodecahedron. Moreover, in the very last proposition, Euclid argues that there are no other regular solids. In this way, this functions like an epilogue to the whole work and provides a justification why the investigation must stop there: simply because there are no other regular solids.

It is not easy to answer with precision the question as to when these figures were discovered; some sources attribute their discovery to the Pythagoreans, and some others attribute the discovery of two of them to Theaetetus. What is particularly interesting is that these solids were used to support one of the first cosmological models.

Plato, in the *Timaeus*, assigns one of the solids to each of the four elements. For several reasons, there is no doubt that Proclus was well aware of Plato's model: first, he was the director of the Academy and thus we assume that he knew his master's work extremely well; second, he prepared an extended commentary on the *Timaeus*, which is extant; third, in his commentary on the *Elements* he often refers to passages in the *Timaeus*, in such a way that reveals a scholarly research. Thus, he had all the material in his mind to form the following connection: Euclid was trying to provide the mathematical foundation of the five cosmic figures, which in their turn provided the cosmological foundation of the cosmos itself. It is worth quoting his thought: 'the whole of the geometer's discourse is obviously concerned with the cosmic figures. It starts from the simple figures and ends with the complexities involved in the structure of the cosmic bodies, establishing each of the figures separately but showing for all of them how they are inscribed to the sphere and the ratios that they have with respect to one another'.

Proclus' idea seems to fit into the general role that is usually ascribed to Plato, as the director and coordinator of research activity in the Academy. This tradition goes back to the well-known story of the Delian problem, preserved by several late commentators . In a summary, the tale goes that the people of Delos, tormented by a plague that Apollo had sent upon them, asked Plato to solve the problem of doubling a cubic altar. According to the Delphic oracle, the plague would leave the island if the Delians succeeded in giving it a solution. So, Plato commissioned the famous 'Academic mathematicians' Archytas, Eudoxus, and Menaechmus to find a solution. In a similar setting, according to Simplicius, Plato set to his earnest students the problem of finding 'what uniform and ordered movement must be assumed to account for the apparent movements of the planets'. According to the tradition, Eudoxus was the first to explain a planet's wandering based on a number of homocentric spheres.



The mathematical purpose of the Elements

A major problem with Proclus' idea is that, besides a weak indication in the *Republic* where Plato notices that solid geometry 'has not been investigated' enough, we have no account that he ever requested such a project.

The actual mathematical purpose of the *Elements* has been a point of conflict among scholars for decades. Based only on the final propositions, Proclus seems to be right: the goal of the *Elements* must have been the construction of the regular solids! On the other hand, various propositions—even whole books of the *Elements*—do not seem to lead towards this goal. Mueller identifies particular propositions in books III, IV, VI, VIII and IX with no future use. In dark colour, notice the dead ends:  The existence of these propositions raises a legitimate question: why does Euclid include other mathematical branches besides geometry in the *Elements*, if his aim was to construct the five regular solids, a seemingly purely geometrical task?

The following table presents the allocation of the first principles in the *Elements*.  We note that the first principles are not spread homogeneously among the books: only book I contains definitions, postulates and common notions all together, whereas four books do not contain any first principles at all; in addition, book X contains definitions scattered in three different parts of the text. This incongruity shows that each single book of the *Elements* must not be considered as a finite work by itself. Thus the *Elements* was probably not compiled in one go, and this is an indication that it was not written to serve one and only one purpose.

Let us have a closer look at the definitions in book XI . Euclid starts by defining what a solid (figure) is (def. XI.1), and continues gradually to define the pyramid (def. XI.12), the cube (def. XI.25), the octahedron (def. XI.26), the icosahedron (def. XI.27), and, finally, the dodecahedron (def. XI.28); at this point, he stops. We notice here a huge gap between the definitions of the pyramid and the cube. This shows that Euclid had other aims as well, one of which, for example, was to study the various kinds of pyramids.

Another fact that weakens Proclus' argumentation is two testimonies, one by Hypsicles that Aristaeus authored a book entitled *Concerning the Comparison of the Five Regular Solids,* and one found in the *Suda* that Theaetetus was the first to 'construct' or 'write about' the five regular solids. Though we are not in place to know the content of Aristaeus' book, the title, at some extend, speaks on itself. And if it is true that mathematical works were written on the regular solids before Euclid, then there is no particular reason to prefer the idea that he was working inside a philosophical tradition; a mathematical tradition existed as well.

In summary, Euclid's mathematical aim in the *Elements* could not have been the mere construction of the regular polyhedra: a) according to the tradition, these were constructed much earlier; b) numerous propositions in the *Elements* appear to have no connection with the construction of the solids; c) we have no such indication from the mathematical tradition (including the *Elements* itself and Euclid's other works). Thus, the study of the regular solids (and not simply their construction)—and I take the middle course here following Artmann—should best be seen as one among the several highlights of the *Elements*. Proclus' idea is influential and romantic in a sense; nevertheless, it is not but a later interpretation of what actually takes place in a collective mathematical work. Euclid was more a part of a new-born and largely independent mathematical tradition than a philosopher.

Heath's arguments

In 1908, Heath stated a viewpoint regarding Euclid's academic background which appears to have become the prevailing belief; namely, that Euclid was trained in Athens by the pupils of Plato, mainly because: a) most geometers who could have taught him were in Athens; b) the people who wrote *Elements* before Euclid were also there; and c) the people on whose works the *Elements* depend were, again, in Athens. Let me briefly examine Heath's arguments.

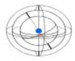
The first premise identifies Athens as the centre of mathematics at the time, and implies that basic rules of probability necessitate that Euclid's mathematical background must be situated there. The truth is that up to 300 BCE, Athens was indeed the intellectual centre of the Greek world; nevertheless, the mathematical activity in other cities was so great that it cannot easily be ignored. Netz records that between 400 and 300 BCE, the cities of Abdera, Colophon, Croton, Cyzicus, Helicon, Lampsacus, Magnesia, Pitane, Samos, Tarentum and Thasos became the place of activity of several memorable mathematicians, like Aristaeus, Hippias, Theudius, and Autolycus. Therefore, we cannot afford to rule out the possibility that Euclid might have studied in any of these places. Furthermore, we have no information about the place of activity of other important mathematicians of this period like Leon, Neoclides, and Antiphon. Given that these people might have travelled around in the Greek world, Euclid, possibly their student, could well have been taught mathematics in any other place besides Athens.

The second premise states that the people who wrote *Elements* before Euclid were in Athens, and implies that since Euclid wrote *Elements* as well, he must have been there. There are three objections to this argument. The first one has to do with our main source on pre-Euclidean *Elements*, Proclus. His reliability in subjects that have to do with the connection of famous people of the past with the Academy is—as I explained earlier—profoundly questionable. The second objection is that the premise itself is not quite true; namely, we have no evidence—including Proclus' testimony—that these people were working in Athens. Proclus, possibly copying from Eudemus, records that three people wrote *Elements* of Geometry before Euclid; namely, Hippocrates of Chios, Leon, and Theudius; of these, only Hippocrates is somehow associated with the Academy. We believe that Theudius lived in Magnesia and we have no indication about Leon's place of activity. The third objection is the fact that Euclid wrote other works besides the *Elements* which appear to have stronger connections with other schools rather than with the Academy. For example, we know by Pappus that Aristaeus wrote *Conics* before Euclid; therefore, a (b)-type argument could lead us to the conclusion that Euclid must have studied with the pupils of Aristaeus (or Aristaeus himself), and not in the Academy. In a sentence, the second premise may lead to Heath's conclusion if we focus only on the *Elements*, and not on any other Euclidean treatises.

The third premise states that the people on whose works the *Elements* depend were in Athens. I do not wish to engage now in detail with the question of the extent to which Euclid incorporated previous research; however, it is well established that most material brought together by Euclid not only in the *Elements,* but in the *Conics*, the *Phaenomena*, and the *Optics* as well, already existed at Plato's time. Some of these propositions were allegedly discovered or proved—or both—in other parts of the Greek world, like Ionia and Magna Graecia. Therefore, it is, again, not true that Euclid incorporates in his works only the mathematical knowledge of Athens.

In summary, this brief discussion shows that there are some serious reasons to question the connection between Euclid and the Academy if we rely only on Heath's arguments. But there are some more arguments which seem to support such a connection and I propose to examine them.

Thinking (only) in circles and lines

According to the tradition, Plato placed a similar to the following inscription over the door of the Academy: 'Let no one unskilled in geometry enter'. Although otherwise implied by the inscription, Plato was notoriously not hospitable to all geometers. Among his most influential views on mathematics was that all proper geometrical figures are composed by the straight (τὸ εὐθύ) and the circular (τὸ στρογγύλον)--a corollary of this idea is that proper geometry should use no other tools but compass and ruler. This view is stated in the *Meno* 74d4-e2, where Socrates and Meno try to discover the definition of figure, in the *Parmenides*, where Parmenides and the young Aristotle try to discover the properties of 'the one'. It is also implied in several other contexts,  from the kind of astronomy he requested from his pupils to the strong aversion he had to the mechanical solutions for the problem of duplicating a cube proposed by Archytas, Eudoxus and Menaechmus. In Plutarch's words 'Plato... inveighed against them for destroying the real excellence of geometry by making it leave the region of pure intellect and come within that of the senses, and become mixed up with bodies which require much base servile labour.' Proclus also records Plato's preference for the two simplest and most fundamental species of line. By virtue of the above, and given that all the constructions involved in the *Elements* require only the use of ruler and compass, the idea may gain support that Euclid was working inside a Platonic context.

To start with, it is correct that all the constructions in the *Elements* are performed via the sole use of ruler and compass. Although this is not specifically stated by Euclid, it is a consequence of postulates 1 and 3; moreover, it is attested by practice. Nevertheless, this cannot establish a bond between Euclid and the Platonic doctrine for one reason: it presents the same fallacy as Heath's second premise. Namely, it makes a conclusion about Euclid's philosophical background only on the basis of what appears in the *Elements*. Nevertheless, we know that Euclid also wrote *Conics* and probably worked on mechanics as well. If we suppose that he was a devoted Platonist, then all of his works should have been compatible with the very basic geometrical principles of Plato. Nevertheless, mechanics and conic sections are included in the 'forbidden' kind of Geometry.

A quest for definitions

In the *Republic*, written around 370 BCE, Plato describes the way mathematical research is conducted: mathematicians are compelled to begin from hypotheses, and, beginning from these, they go through the rest and end consistently with that which they set out to examine. It is not clear what Plato meant by hypotheses; nevertheless, he provides three examples, i.e. the odd, the even, the figures, all of which are included in Euclid's first principles. Thus, on a more general level, Plato's description presents some similarities with the deductive plan laid down in the *Elements*; moreover, some of Euclid's first principles appear in the Platonic corpus. I propose to examine whether this are enough to establish a Platonic influence on the formation of Euclid's *Elements*; however, before doing so, let me distinguish some characteristics of Euclid's definitions.

What constituted a definition for Euclid appears to be open to interpretation, as there are at least three different kinds of definitions in the *Elements*. First, we have definitions of elementary mathematical objects, like points, lines, and units; see, for example, I.1 and VII.1. Second, we have definitions of advanced mathematical objects derived from the former ones; for example, see I.11. Third, we have propositions describing the relationship between objects already defined. For example, see I.3 and I.6.

All examples of Plato's hypotheses in the aforementioned passage in the *Republic* belong to the second kind of Euclid's definitions. Thus, Plato appears to give an account of a pre-Euclidean practice in which mathematical treatises contained only definitions of this kind. At the same time, in his writings, Plato applies definitions of all three kinds. For example, in three of Plato's dialogues we have descriptions similar to Euclid's def. VII.1 (1st kind), in the *Parmenides* we have a description similar to def. I.4 (2nd kind), and in the *Meno* we have a definition of 'shape' as 'the limit of a solid' followed by a definition of 'limit' in the *Parmenides* (3rd kind). This divergence, could lead to the idea that the appearance of definitions of the first kind in Euclid might have been the result of Platonic criticism, a view also supported by Mueller's view that definitions of the first kind are never invoked in Euclid's proofs. This is an indication that they were not added to serve a mathematical purpose, but perhaps as a response to philosophical criticism.

Let me examine these views, starting with Mueller's idea that definitions of the first kind are not invoked in Euclid's proofs.  Let me go to Euclid's proposition I.1 at the point where he draws the circles BCD and ACE. His next step is to join points A, C and points C, B, and construct the straight line segments AC and BC. Although not explicitly mentioned here—after all Euclid never cites the first principles he uses—definition I.1 is engaged in this step through postulate 1. More particularly, according to this postulate, we are allowed to draw a straight line from any point to any point. But this may happen only because points do not have parts. If that was not the case, the straight line would pass through a part of point A, and joint with a part of point Γ. Thus, definitions of the first kind do serve a mathematical purpose in the *Elements* of equal importance as the other kinds of definitions.

Now let me examine the idea that elementary definitions in the *Elements* might have been the result of Platonic criticism. According to this view, the definitions in mathematical treatises before Plato's time would have been a collection of interconnected definitions like, 'points are the extremities of a line'. But, evidently, that was not the case. In a passage in the *Topics*, Aristotle refers to the way his contemporary mathematicians defined their subject: speaking about points, lines, and surfaces, he claims that all define the prior by means of the posterior; moreover, in the *Metaphysics*, he proposes a definition for part in a way similar to Euclid. This shows that mathematicians of Aristotle's generation were not influenced by Plato's definitions---at least in this way. Thus, it is unlikely that Euclid, around three generations after Plato was directly influenced by him, when older mathematicians were not.

Another argument against this view is that definitions of elementary mathematical objects appear much earlier than Plato. Aristotle tells us that the Pythagoreans were the first to define the essence of objects and that Archytas investigated general definitions. Nicomachus also gives a definition of even number and claims that it is Pythagorean. Also note Euclid's def. VII.1: According to Taisbak, this definition has an 'unmistakable Platonic ring; namely, a thing is called one by virtue of its participation in the idea of —unity'. On the other hand, Pritchard maintains that the definition owes nothing to Platonic metaphysics, simply because it does not originate with Plato. Sextus Empiricus attributes to Pythagoras the following saying: 'a first principle of beings is the unit, by participating in which each of the beings is called one'.

One may not deny that there are some similarities between Euclidean mathematics and mathematics of Plato's time. Nevertheless, the differences are too strong to be dismissed. To give an example, in *Posterior Analytics* Aristotle records a proof of the proposition: 'the angles of the base of the isosceles triangles are equal'. Although, the same proposition appears in the *Elements* as I.15, the reasoning of the Aristotelian proof is not at all similar. But there are more differences: in the *Parmenides*, 'round' is defined as 'that which the furthest points in all directions are at the same distance from the middle point' and the sphere is similarly defined in the *Timaeus*. However, in the *Elements*, the sphere is defined differently: 'When, the diameter of a semicircle remaining fixed, the semicircle is carried round and restored again to the same position from which it began to be moved, the figure so comprehended is a sphere'. In addition, despite having common notions in the Platonic corpus, we do not find any postulates. These differences are crucial because they are two-fold. Besides the apparent mismatch, they imply a difference in terminology. That is to say, what Plato calls 'hypotheses' Euclid would call, depending on the case, 'definition', 'postulate', or 'common notion'. A difference in terminology always argues against the possibility of direct influence.

Finally, let us notice that the mere existence of similarities—even numerous ones (which is not the case, by the way!)—cannot reveal the direction of the influence. This very interesting subject has been the main point of a famous debate between Knorr and Szabó, and, at the moment, it seems equally probable that Plato, Aristotle, and Euclid were drawing from a common mathematical tradition. In other words, the similarities in their work cannot be used as an argument in favour of the view that Euclid belonged to any philosophical school. In conclusion, Plato's description of mathematical practice in the *Republic* presents some similarities with what takes place in the *Elements*. It is also true that some of Euclid’s first principles appear in Plato’s writings. Nevertheless, this is not enough to establish a direct Platonic influence on the formation of the *Elements*.

Revisiting Euclid’s agenda

According to the tradition, Euclid wrote several treatises, most of which are extant. The lengthy catalogue comprises: the *Elements* of geometry, the *Elements* of music, the *Data*, the *Pseudaria*, the *Porisms*, the *Conics*, the *Phaenomena*, the *Optics*, the book on *Divisions*, the *Surface-loci*, the *Book of the Balance*, and the *Book on the Heavy and Light*. Although we have no direct evidence regarding Euclid’s agenda, historians agree that the character of his works appear to be introductory and collective. The ancient scholars shared a similar point of view: Marinus writes: ‘for almost every mathematical science, he placed before *Elements* as introductions, like for the whole of geometry in the 13 books, and for astronomy in the *Phaenomena*, and for music and optics, similarly, he handed down *Elements*. Proclus is more specific: ‘[Euclid] brought together the *Elements*, collected many of Eudoxus’ (results), perfected many of Theaetetus’, and also brought to irrefutable demonstrations those that had been rather loosely proved by his predecessors.

Now, regarding Euclid’s institutional background, we have almost no information. Nevertheless, for numerous reasons, there is a wide-spread opinion among historians that Euclid taught mathematics in Alexandria, and was the foremost in the long chain of mathematicians that appear to have worked in the city. Acerbi has recently questioned the ‘mito della formazione scientifica ad Alessandria’, and argued that this is nothing but a retrojection of our modern educational system. Thus, I propose to re-examine the possibility that Euclid taught mathematics, based on Lloyd and Sivin’s criteria of what constitutes a school in Euclid’s time.

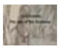
According to these scholars, the existence of a school is determined by two factors: (a) the existence of (at least one) teacher and students; and (b) the existence of a shared doctrine. Two accounts directly support the idea that Euclid had students: (a) Pappus’ testimony that Apollonius spent much time studying with the pupils of Euclid; and, (b) Stobaeus’ anecdote that ‘someone who had begun geometry with Euclid, when he learned the first theorem, asked Euclid, ‘What shall I earn by learning these?’ And Euclid called the slave and said, ‘Give him three obols, because he must gain out of whatever he learns.’

Regarding the existence of a common doctrine in Euclid's time, the evidence is circumstantial, but cumulative. Archimedes and Apollonius, who flourish some years after Euclid, appear to know his work. Archimedes does not write *Elements* (apparently, because a version of the *Elements* is already in circulation), and appears to continue Euclid's investigation on solid figures. Moreover, the Arabic tradition associates Archimedes' mechanical investigation with Euclid. In addition, his correspondence with Eratosthenes and Conon advocates in favour of the view that he spent a part of his life in Alexandria, and, thus, it reveals the existence of a mathematical community in the city; namely, a group of people sharing a common knowledge, practice and interests. Acerbi correctly notes that Archimedes' prefaces are private letters, and do not imply an institutional setting. Nevertheless, even if we assume that Archimedes never spent time in Alexandria, and that he addressed Eratosthenes only because of his institutional position, this can also be seen as evidence of the presence of a mathematical tradition in Alexandria around the director of the institution, worthy of receiving, understanding, and replying to these letters. Finally, let me note some information usually neglected by historians who assume that the only evidence of relationship between Archimedes and Eratosthenes is Archimedes' prefaces. According to Proclus, Eratosthenes also records that he was contemporary of Archimedes.

The idea of a common tradition is also supported by Netz's (1999) study, who argues that Archimedes, Euclid, and Apollonius shared common practices like the use of the lettered diagram, limited and specialised lexicon, discursive conventions, and criteria of validity. In addition, Cuomo notes that 'a rather large, if not entirely unnamed, public for mathematics emerges also from the works of Archimedes and Apollonius'. Among the people Archimedes mentions are Zenodorus, Pythion of Thasus, Conon and Dositheus. Hypsicles also addresses the so-called book 14 of the *Elements* to Protarchus and mentions Basilides of Tyre, who shared a passion for mathematics with Hypsicles' father.

Returning to Acerbi's views, let me note that they are largely based on Netz (1997) overall picture of Greek mathematicians as a rather loose community and very few in numbers. Let me also note that Netz's results are founded upon his own definition of Greek mathematicians, according to which the discovery of an original demonstration is a prerequisite. But a school of mathematics apparently does not consist only of researchers, but of teachers of mathematics and students as well. Obviously, if we rely on this definition, we will not be able to see—as Acerbi did not—any school of mathematics, but rather a fragmentary picture of isolated gifted discoverers. This case shows that the quest for definitions in the historiography of Greek mathematics is not a mere exercise of scholarly esoterica, but an essential prism, which shapes our image of our mathematical past.

By virtue of the above, it stands to reason to assume that several of Euclid's works—if not all—were also used for teaching purposes. Euclid probably taught mathematics in Alexandria, and, if we are to speak in Lloyd and Sivin's terms, he founded a school in the city. The ancient scholars shared the same opinion: Marinus hints at the usefulness of the *Data* in teaching, and Proclus praises the educational value of the *Elements*. Moreover, describing the, now lost, *Pseudaria*, Proclus records that it contained a collection of false proofs and methods to help beginners avoid fallacies in geometry. Finally, the prefaces to Heron's *Definitions* and Diophantus' *Arithmetica* claim that both: (a) were framed in terms of pedagogy, and (b) followed Euclid's methodology.



Conclusions

In conclusion, Euclid was not a philosopher. We have no information that he belonged to any philosophical tradition of his time, and his mathematical treatises do not contain philosophical arguments. Proclus' idea that Euclid composed the *Elements* just to provide a foundation for Plato's cosmological model is undoubtedly romantic; nevertheless, it cannot be established upon the deductive structure of the *Elements*, and thus it is better seen as a part of a great Platonically biased tradition, of which Proclus was a member. The fact that Euclid was not a philosopher does not exclude the possibility that he incorporated in his works a number of treatises which might have been written by previous mathematicians in order to serve Proclus' idea—Eudoxus is of course the number one suspect in this case; nevertheless, as a whole, the *Elements* was not written to serve the Platonic cosmic theory.

One of the most interesting questions around the origin of Greek mathematics is related to the extent to which its development was connected with the development of Greek philosophy at the time. The famous Knorr-Szabó debate has coined the two opposite views on this subject, with the one side claiming that it was mathematics that affected philosophical argumentation, and the other side supporting the opposite direction of influence. Due to irrecoverable documental evidence, it comes to no surprise that a *terminus a quo* could not be decided; nevertheless, this research concludes that a relative *terminus ad quem* can be pinpointed in terms of time, around Euclid's time.

Evidently, traces of independence from philosophical schools can be found before Euclid, and, on the other side, traces of dependence from philosophical schools can be found after Euclid. Nevertheless, Euclid's works offer a glimpse into a historical period in which mathematicians start to (a) resort to the works of previous mathematicians in order to compile their own works; and (b) do not write about other subjects except for mathematics.

To conclude, let me return to the initial question which sparked this presentation; namely, was Euclid a child of the Academy, despite the fact that he did not belong to the so-called Platonic circle? The answer is yes! The emergence of small mathematical communities, the growth of mathematical knowledge, the development of common language, terminology, and apodictical methodologies, are only some of the factors that promoted the autonomy of the field. And all these were developed in Plato's Academy. A final remark: this presentation can also be seen as an opportunity to consider another polarity found in Rafael's fresco.

Thank you very much.