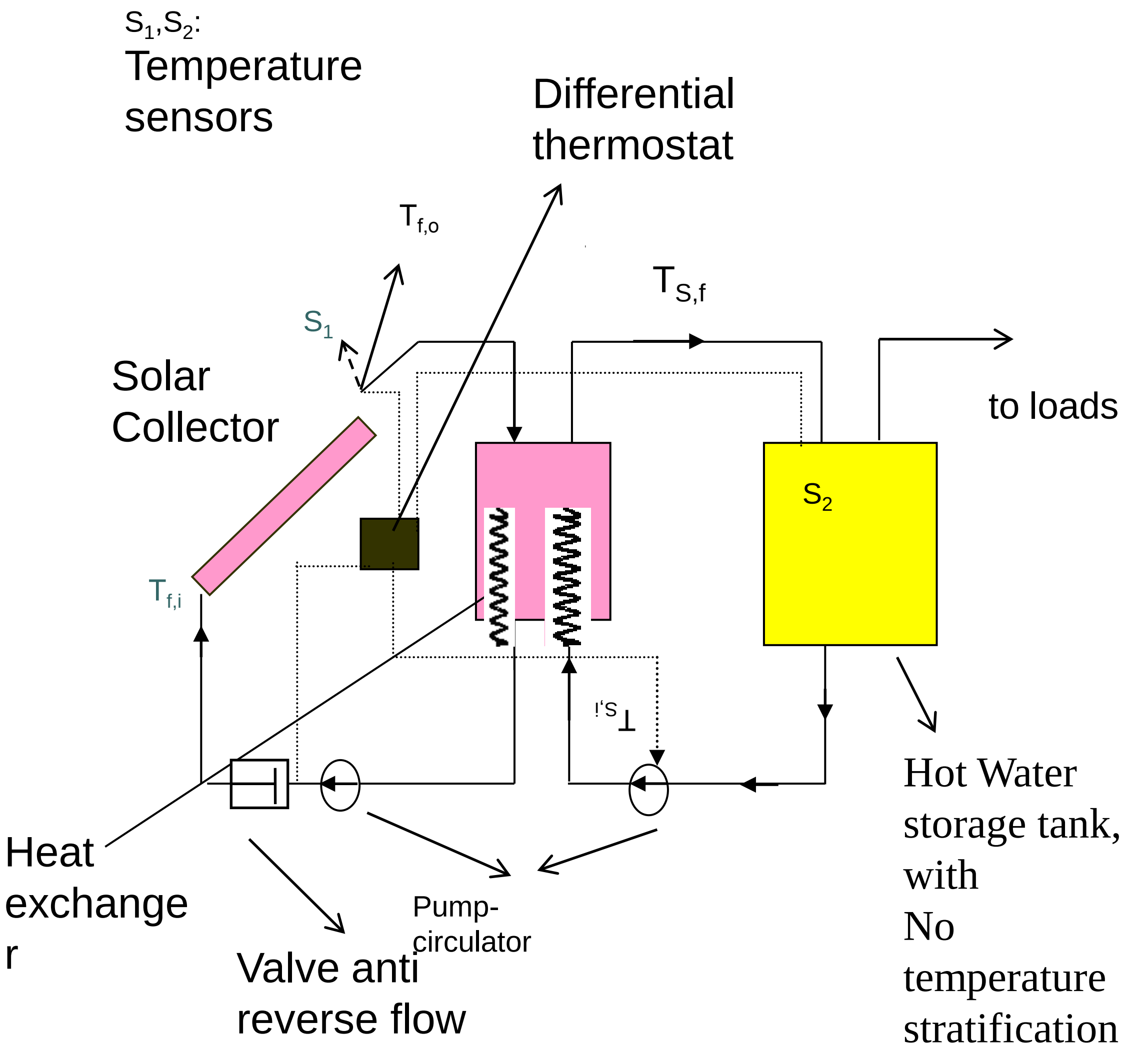
Basics of thermal analysis of solar collector

Simulation of operation of solar systems to determine the Thermal Gain.



The useful thermal energy Qu can be calculated from the relation:

$$Q_u = (m C_p)_f (T_{f,o} - T_{f,i}) \qquad (1.1)$$

or normalized to solar collector surface, the thermal power output is given by:

$$\dot{Q}_u = (\dot{m} C_p)_f (T_{f,o} - T_{f,i}) \qquad (1.2)$$

It can be also shown that the useful thermal gain from a solar collector is given by:

$$\dot{Q}_u = F_R A_c [I_T (\tau\alpha) - U_L (T_{f,i} - T_\alpha)] \qquad (1.3)$$

A similarity to the previous expressions holds for the hot water tank,too:

$$\dot{Q}_u = (\dot{m} C_p)_s \cdot \left(T_{s,f} - T_{s,i}\right) \qquad (1.4)$$

or equivalently

$$\dot{Q}_u = (\dot{m} C_p)_{min} \cdot \varepsilon \cdot \left(T_{f,o} - T_{s,i}\right) \qquad (1.5)$$

The **coefficient of effectiveness**, ε, is given by an expression in any Heat Transfer book,

$$\varepsilon = \frac{Q}{Q_{max}} = \frac{(\dot{m} C_p)_c (T_{f,o} - T_{f,i})}{(\dot{m} C_p)_{min} (T_{f,o} - T_{s,i})} = \frac{1 - e^{-NTU(1-C)}}{1 - C \cdot e^{-NTU(1-C)}} \qquad (1.6)$$

From the expression (1.2) of calorimetry one gets:

$$C = (\dot{m} C_p)_{min} / (\dot{m} C_p)_{max} \qquad NTU = (U_A \cdot A)_{\varepsilon\nu} / (\dot{m} C_p)_{min}$$

$$T_{f,o} = T_{f,i} + \frac{\dot{Q}_u}{(\dot{m} C_p)_f} \qquad (1.7)$$

Substitute $T_{f,i}$ from (1.7) to (1.3), then the equation which provides the Thermal Power stored in the system-tank takes the form:

$$\dot{Q}_u = \frac{A_c F_R \left[I_T(\tau\alpha) - U_L\left(T_{f,o} - T_\alpha\right)\right]}{1 - \frac{A_c F_R U_L}{(\dot{m} C_p)_f}} \tag{1.8}$$

We solve eq. (1.5) for $T_{f,o}$ and we get:

$$T_{f,o} = \frac{\dot{Q}_u}{(\dot{m} C_p)_{min} \cdot \varepsilon} + T_{s,i} \tag{1.9}$$

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References

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Substitution of $T_{f,o}$ to (.1.8), gives:

$$\dot{Q}_u = \frac{A_c F_R \left[I_T(\tau\alpha) - U_L \cdot \left[\frac{\dot{Q}_u}{(\dot{m}C_p)_{min} \cdot \varepsilon} + T_{s,i} - T_\alpha \right] \right]}{1 - \frac{A_c \cdot F_R U_L}{(\dot{m}C_p)_f}} \quad (1.10)$$

The expression (1.10) can be easily simplified to:

$$\dot{Q}_u = \frac{A_c F_R \left[I_T(\tau\alpha) - U_L \left(T_{s,i} - T_\alpha \right) \right]}{1 + \frac{A_c F_R U_L}{(\dot{m}C_p)_f} \times \left[\frac{(\dot{m}C_p)_f}{(\dot{m}C_p)_{min} \cdot \varepsilon} - 1 \right]} \quad (1.11)$$

We define a new parameter, F´R:

$$F'_R = \frac{F_R}{1 + \frac{A_c F_R U_L}{(\dot{m} C_p)_f} \times \left[\frac{(\dot{m} C_p)_f}{(\dot{m} C_p)_{min} \cdot \varepsilon} - 1 \right]} \quad (1.12)$$

Hence, the expression (1.11) is simplified as to:

$$\dot{Q}_u = A_c * F_R' \left[I_T (\tau\alpha) - U_L \left(T_{s,i} - T_\alpha \right) \right] = F_R A_c * \frac{F_R'}{F_R} \left[I_T (\tau\alpha) - U_L \left(T_{s,i} - T_\alpha \right) \right] \quad (1.13)$$

Let us consider a small time period the solar collector system operates. Then, the mean water temperature in the storage tank is determined by:

$$T_s = \frac{T_{s,i} + T_{s,f}}{2} \tag{1.14}$$

The heat delivered by a collector, Ac, in a period, Δτ, to the tank may be determined by:

$$Q_{u,\Delta\tau} = (MC_p)_s(T_{s,f} - T_{s,i}) = \int_{\tau}^{\tau+\Delta\tau} \dot{Q}_u dt \tag{1.15}$$

We divide both sides of (.1.15) over Ac to normalize the expression. Then

$$q_{u,\Delta\tau} = (mC_p)_s(T_{s,f} - T_{s,i}) \tag{1.16}$$

or equivalently

$$T_{s,f} - T_{s.i} = q_{u,\Delta\tau}/(mC_P)_s \tag{1.17}$$

Substitute Ts,f from (1.17) to (1.14). We get:

$$T_s = T_{s,i} + \frac{q_{u,\Delta\tau}}{(2mC_P)_s} \tag{1.18}$$

T_s is the mean temperature of the water in the tank in the above time interval. Integration of (1.13 or 1.15) for this time period gives:

$$Q_{u,\Delta\tau} = F_R{}' \times A_c[H_n\overline{(\tau\alpha)} - U_L(T_s - \bar{T}_\alpha)\Delta\tau] \tag{1.19}$$

Substitute Ts from (1.18) to (1.19). We get:

$$\mathbf{q_{u,\Delta\tau}} = \frac{\mathbf{F_R{}'[H_n(\overline{\tau\alpha}) - U_L(T_{s,i} - \overline{T_\alpha})\Delta\tau}}{\mathbf{1 + \dfrac{F_R{}'U_L}{2(mC_P)_s} \times \Delta\tau}} \tag{1.20}$$

Let us analyze a real case

A Solar Collector System, as the one shown in the 1st figure, has parameters: $F`_R U_L$=3.5 W/m^2*K and $F`_R(\tau\alpha)_n$= 0.69

and is placed at horizontal position in Pygros.

The storage tank has capacity 50 l/m^2.

Let the storage tank temperature at 7:30 be 20 C.

Please determine the hourly temperature in the tank and the hourly efficiency.

Data input: ambient temperature , T_a, and the mean hourly global solar radiation, H_n. Values are given in the Table below

Data input and values of basic quantities as provided by the iteration procedure to be outlined below.

Time **18.4.1999**	**$T_α$ ºC**	$H_n\ (\frac{kJ}{m^2})$	$q_u\ (\frac{kJ}{m^2})$	**$T_{s,i}$ ºC**	**$T_{s,f}$ ºC**	**$N=q_u/H_n$**
7:30 – 8:30	**15.0**	**720**	**421**	**20**	**22**	**0.58**
8:30 – 9:30	**15.5**	**1476**	**909**	**22**	**26**	**0.61**
9:30 – 10:30	**16.5**	**1980**	**1206**	**26**	**32**	**0.60**
10:30 – 11:30	**17.0**	**2484**	**1479**	**32**	**39**	**0.59**
11:30 – 12:30	**17.5**	**2844**	**1640**	**39**	**46**	**0.57**
12:30 – 13:30	**18.0**	**3240**	**1816**	**46**	**55**	**0.56**
13:30 – 14:30	**19.0**	**3250**	**1729**	**55**	**63**	**0.53**
14:30 – 15:30	**19.0**	**2968**	**1439**	**63**	**70**	**0.48**
15:30 – 16:30	**18.0**	**2412**	**971**	**70**	**75**	**0.40**
16:30 – 17:30	**17.5**	**1800**	**498**	**75**	**77**	**0.27**
17:30 – 18:30	**17.0**	**1210**	**68**	**77**	**78**	**0.05**

To determine the quantities $T_{s,f}$, $T_{s.i}$

A. Time Interval 7:30 – 8:30 am

Step 1st : We determine the normalized useful heat by (1.20).

$$\mathbf{q_{u,\Delta\tau}} = \frac{720\,\text{kJ}/\text{m}^2 \times 0.69 - 3.5\,\dfrac{\text{W}}{\text{m}^2\text{K}} \times (20^0 - 15^0) \times 3600\text{s}}{[1 + \dfrac{3.5\,\dfrac{\text{W}}{\text{m}^2 K} \times 3600\text{s}}{2 \times 50\,\dfrac{\text{kg}}{\text{m}^2} \times 4180\,\dfrac{\text{J}}{\text{kg K}}}] = 1.03} = \mathbf{421}\,\text{kJ}/\text{m}^2 \qquad (1.21)$$

Step 2nd : We determine tank temperature, $T_{s,f}$, at the end of the 1st interval 8:30 amusing expression (1.17)

$$T_{s,f} = 20^0 C + \frac{421{,}000 \text{ J/m}^2}{50 \text{ kg/m}^2 \times 4180 \text{ J/kg m}^2} = \mathbf{22}\ ^0C \qquad (1.22)$$

Step 3rd : Determine efficiency, η, during this short period by :

$$\eta = \frac{Q_{u,\Delta\tau}}{A_c H_n} = \frac{q_{u,\Delta\tau}}{H_n} \qquad \eta = \frac{q_{u,\Delta\tau}}{H_n} = \frac{421 \text{ kJ / m}^2}{720 \text{ kJ / m}^2} = \mathbf{0.58} \qquad (1.23)$$

B. Time Interval 8:30 – 9:30am

We follow the same procedure as before. We put for this period 8:30-9:30 as $T_{s,i}$, the $T_{s,f}$ value of the previous interval.

Determine $\mathbf{q_{u,\Delta\tau}}$ from (1.20)

$$\mathbf{q_{u,\Delta\tau}} = \frac{1476\,\mathrm{kJ/m^2} \times 0.69 - 3.5\dfrac{\mathrm{W}}{\mathrm{m^2\,{}^0K}} \times (22^0 - 15.5^0) \times 3600\mathrm{s}}{1.03} = \mathbf{909}\,\mathrm{kJ/m^2}$$

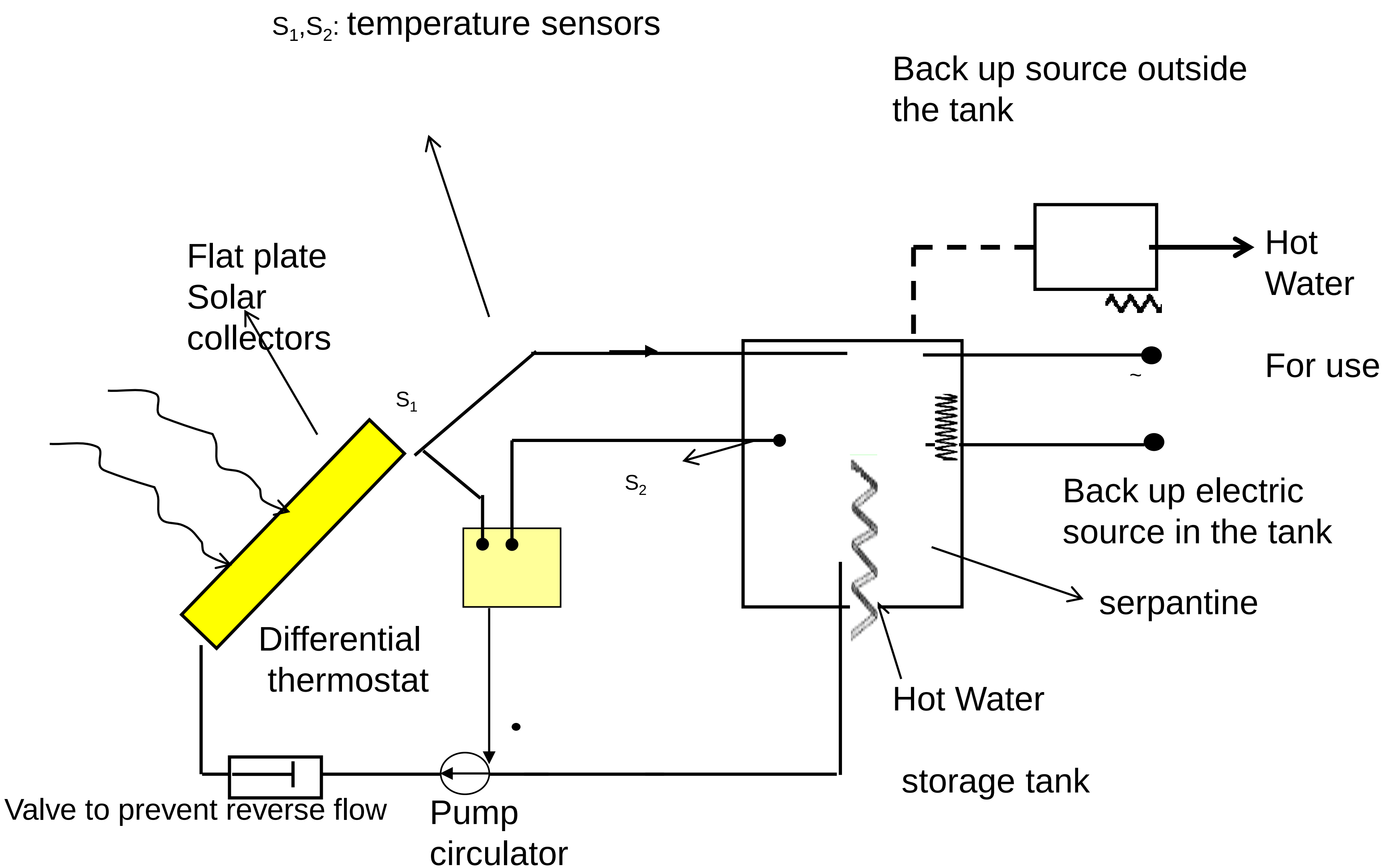
(1.24)

Determine $T_{s,f}$ from (1.17)

$$\mathbf{T_{s,f}} = 22^0\mathrm{C} + \frac{909000\ \frac{\mathrm{J}}{\mathrm{m}^2}}{50\ \frac{\mathrm{kg}}{\mathrm{m}^2} \times 4180\ \frac{\mathrm{J}}{\mathrm{kg}^o\mathrm{C}}} = \mathbf{26}\ ^0\mathrm{C} \tag{1.25}$$

Then, the efficiency is estimated by:

$$\boldsymbol{\eta} = \frac{\mathbf{q_{u,\Delta\tau}}}{\mathbf{H_n}} = \frac{\mathbf{909\,kJ / m^2}}{\mathbf{1476\,kJ / m^2}} = \mathbf{0.61} \tag{1.26}$$


S1,S2: temperature sensors
Back up source outside the tank
Hot Water
Flat plate Solar collectors
For use
S1
S2
Back up electric source in the tank
serpantine
Differential thermostat
Hot Water
storage tank
Valve to prevent reverse flow
Pump circulator

2. **A generalized analysis to consider the Load,** too.

Let us consider q_s as the net stored heat normalized to collector surface; that is when the Load, Q_L, is subtracted. Correspondingly, the thermal load per collector surface is denoted by, ($q_L = Q_L / Ac$). Then it holds :

$$q_S = q_{u,\Delta\tau} - q_L \qquad q_{u,\Delta\tau} = q_S + q_L \tag{2.1}$$

Following the procedure as for expression (1.18) there is given that:

$$T_s = T_{s,i} + \frac{q_s}{2(m\,C_P)_s} \tag{2.2}$$

2. A generalized analysis to consider the Load, too.

Substitute Ts to (1.19). Then, the expression (1.20) is modified and due to (2.1) we get :

$$\mathbf{q_s} = \frac{\mathbf{F'_R}\left[\mathbf{H_n}(\overline{\boldsymbol{\tau\alpha}}) - \mathbf{U_L}(\mathbf{T_{s,i}} - \overline{\mathbf{T_\alpha}})\Delta\tau\right] - \mathbf{q_L}}{1 + \dfrac{\mathbf{F'_R U_L}}{2(\mathbf{m C_p})_s}\boldsymbol{\Delta\tau}} \quad (2.3)$$

We substitute (2.3) in (2.1) and we finally get the generalized iterative formula:

$$\mathbf{q_{u,\Delta\tau}} = \frac{\mathbf{F'_R}[\mathbf{H_n}(\overline{\boldsymbol{\tau\alpha}}) - \mathbf{U_L}(\mathbf{T_{s,i}} - \overline{\mathbf{T\alpha}})\Delta\tau}{1 + \dfrac{\mathbf{F'_R U_L}}{2(\mathbf{m C_p})_s}\boldsymbol{\Delta\tau}} + \frac{\mathbf{q_L}}{1 + \dfrac{2(\mathbf{m C_p})_s}{\mathbf{F'_R U_L}\boldsymbol{\Delta\tau}}} \quad (2.4)$$

References

Renewable Energy Systems: Theory and Intelligent Applications
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Nova Science Publishers, N.Y., 2012