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FINAL REPORT ON TEXTBOOKS ANALYSIS

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SH4_3510 "EDARCPADSS"

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Epistemological and Didactical Aspects Related to the Concept of Periodicity Across Different School Subjects.

(Ελληνικά) **Επιστημολογικές και διδακτικές απόψεις σχετικές με την έννοια της περιοδικότητας σε διαφορετικά σχολικά μαθήματα.**

Φορέας Υποδοχής

ΑΝΩΤΑΤΗ ΣΧΟΛΗ ΠΑΙΔΑΓΩΓΙΚΗΣ ΚΑΙ ΤΕΧΝΟΛΟΓΙΚΗΣ ΕΚΠΑΙΔΕΥΣΗΣ (Α.Σ.ΠΑΙ.Τ.Ε.)

Ερευνητική μονάδα φορέα υποδοχής

ΓΕΝΙΚΟ ΤΜΗΜΑ ΠΑΙΔΑΓΩΓΙΚΩΝ ΜΑΘΗΜΑΤΩΝ (ΓΕΤΠΜΑ)

Στοιχεία Μεταδιδάκτορος Ερευνήτριας

ΤΡΙΑΝΤΑΦΥΛΛΟΥ-ΚΑΛΛΙΩΡΑ ΧΡΥΣΑΥΓΗ

Στοιχεία Επιστημονικής Υπευθύνου

ΣΠΗΛΙΩΤΟΠΟΥΛΟΥ ΒΑΣΙΛΙΚΗ, Καθηγήτρια

Chapter 1: INTRODUCTION

The overall aim of the research project

The work presented in this report is part of a research project that aims to identify epistemological and didactical aspects among different educational practices concerning the concept of "periodicity". Periodicity is an essential scientific concept because it plays a central role in the school curriculum and is expressed in different educational fields where it acquires practical importance.

The students form this concept in different school subjects like science, mathematics, applied technology and astronomy. The above subjects are recognized as human, cultural and historically determined activities (Vygotsky, 1981; Leont'ev, 1978). This means that the presence of the concept of periodicity in the school curriculum cannot be understood or analyzed, without reviewing the practices adopted in these communities.

Furthermore, understanding periodicity demands connections between the periodic phenomena of everyday life and the natural world on the one hand and the abstract mathematical notions which model them on the other hand.

The role of textbooks

The study of mathematical and pedagogical practices is important as these influence students' conceptions. Two factors are considered critical for the formation of students' pedagogical practices: the textbooks used and the teachers' cognitive and didactical knowledge. In most countries (Greece included) textbooks are used by teachers as the main source for their classroom activities.

In the present study we consider the textbooks as the mediating tools in an Activity Theory perspective (Vygotsky, 1981).

Researchers argue that students facing difficulties in handling and integrating the abstract descriptions l (e.g. mathematical models) and the perceptual (e.g. the periodic motion of a pendulum) aspects of periodicity (Buendia & Cordero, 2005). The question of how textbooks support this difficult integration of the issues above is crucial and open (Dreyfous & Eiseberg, 1980).

The role of textbooks' argumentation

Love and Pimm (1996) highlight the role of argumentation developed in the school textbooks in the meaning-making process by denoting that although the implied relation between the reader and the text is inherently passive*, "the most active invitation to any reader seems to be working through the text to see why the particular 'this' is so"* (p. 371). In this direction, in the science context, Chi and her colleagues (Chi, deLeeuw, Chiu & LaVancher, 1994) point out that students generate self-explanations in order to fill in substantial details in texts in school textbooks. In spite of the pervasive presence of textbooks in educative practices, few research studies have focused on textbook analysis in relation to the logic of the presented knowledge. These studies identify empirical inductions and deductions as modes of reasoning (Stacey & Vincent, 2009) while many times these arguments occur presumably in conjunction with achieving a better understanding (Cabassut, 2005). Roseman, Stern & Koppal (2010) contend that in order to be considered high-quality, textbooks must be coherent and help students make the connections necessary to organize their new knowledge into a meaningful whole. More to the point, while a

The argumentation developed in school texts in different subjects for a common topic is rather limited. Analysing the argumentation produced in this case is didactically important since these texts are addressed to the same student who is ‘responsible’ for making the appropriate conceptual connections.

The role of visual features in texts

The importance of the multimodality approach has been argued by Kress & van Leeuwen (2006). They consider that visual and verbal elements in texts are two items of information which are interrelated as follows: by elaboration (when an item elaborates on the meaning of another by further specifying or describing it) and by extension (when an item extends the meaning of another by adding something new to it). In this study, the function of the visual representations in the reasoning process is investigated thoroughly. Moreover, in the analysis of textbooks argumentation not only the outcomes of inductions and deductions (i.e. the general conclusions of reasoning) but also the examples and images provided in the text are also important on the concept image formation (Tall & Vincent, 1981). In this direction, Biehler (2005) also considers that the representations available for working with are essential elements constitutive of the meaning of any mathematical concept.

Hence, the visual features in understanding aspects of periodicity are crucial and worth studying since they have a considerable effect on the way students experience and build new concepts in the field. So, in this study we explore the visual as well the verbal components of the text as well the co-deployment of visual representations (VRs) and reasoning.

The role of the proposed exercises

Since the students are trained to learn through solving exercises and problems we decided to analyze the sets of the proposed exercises in the specific chapters. Our research question on this dimension is: What are the conceptions of periodicity that may be stimulated by the solutions to exercises and problems in the given sample?

The aim of the present report

Our general aim in this phase is to study the flow and continuity of knowledge in mathematical and related referential textbooks in general and technical secondary education on the notion of periodicity (visual representations; explanations; issues controlling students’ knowledge) and trying to identify conceptual ruptures, gaps and overlaps associated with this concept.

Our specific questions are:

What type of conceptual aspects of periodicity introduced and supported in school texts?

What is the nature of argumentation that is employed in the textbooks to support the conceptualization of periodicity?

What is the relation between argumentation and visual representations in different school subjects?

What is the role of visual representations in the developed new knowledge?

Is there continuity between the presented knowledge and the proposed exercises?

What other types of epistemological and didactical issues raised from our analysis?

Outline of this report

In this phase Greek textbooks of mathematics, physics, astronomy and applied technologies from grade 9 to grade 12 of both Unified Upper Secondary Schools and Technical Vocational Educational Schools (EPAL) are studied. The sample consists of all the chapters where the notion of periodicity is explicit or implicit.

The theoretical framework will be presented in chapter 2 and in chapter 3 the methodology applied. In Chapter 4 the results of the qualitative and quantitative analysis will be presented and issues that emerged from this analysis will be highlighted. Finally, in Chapter 5 we present the general conclusions and issues we want to research in depth in the next phases.

Chapter 2: THEORETICAL FRAMEWORK

Vergnaud's (2009) theory of conceptual fields

We adopt Vergnaud's (2009) theory of conceptual fields that addresses the process of conceptualization of reality. It is a pragmatic theory as it presupposes that knowledge acquisition is shaped by situations, problems and actions for the subject. It is, therefore, through the situations that a concept acquires meaning for a student. Vergnaud considers that a concept is a triplet of a set: **C=(S, I, L)** where **S** stands for the set of situations which give sense to a concept (*the referent*); **I** stands for the set of operational invariants associated with the concept (*the meaning*); **L** stands for the set of linguistic and non-linguistic representations which allow for the symbolic representation of a concept, its attributes, the situation to which it applies and the procedures it nourishes. In this paper, the thematic units where the concept of periodicity appears in school texts are considered as situations (S); the patterns of argumentation developed by the author of the textbooks in these units as operational invariants (I) as well as the rules that generate the reasoning activity for the establishment of new knowledge; the tools employed by the author in the argumentation process as linguistic and non-linguistic representations (L).

Substantial argumentation

Since our interest is on argumentation techniques or methods used in textbooks to reason about the presented new knowledge we are interested in what Toulmin (1969) calls 'substantial argumentation' (p. 234). Substantial argumentation does not have the logical stringency of formal deductions but is used for gradual support of different statements. Toulmin establishes the importance of practical arguments and their logical canons, which may not be entirely safe as formal mathematical arguments, but are necessary tools of thinking in general. Argumentation here is taken to mean the use of reasoning for the construction of knowledge presented in a text for the purpose of convincing the students of the truth of a conclusion. This is considered to affect the students' ways of understanding and conceptualizing the field. We expect to identify elements of the invariant notion of periodicity across subjects by analysing the co-deployment of linguistic (i.e. verbal) and non-linguistic (i.e. visual) components in the argumentation developed in school texts in specific situations (i.e. thematic units) related to periodicity. Argumentation here is taken to mean the use of different modes of reasoning with the purpose of convincing the students for the truth of a conclusion.

Argumentation and reasoning in different contexts

Daily life reasoning is characterized as *informal* since people draw inferences from uncertain premises. Scientific reasoning may be either *deductions* based on a set of a priori premises; or *inductive* generalizations based on laws; or inferences to the best explanation as in Darwin's development of evolutionary theory (Szu & Osborne, 2012). Furthermore, the modes of reasoning in science text could be (a) *logical* when they are based on the finished products of science (i.e. laws, principles, models, theories and mathematical and algorithmic procedures); and (b*) evidential* when they are based on experiments and intuitiveness. On the same direction mathematical reasoning on school texts can be categorized as (a) *deductive* (by using a model, or a specific or a general case); (b) *empirical* reasoning (e.g. experimental demonstration); and (c) metaphorical reasoning (Stacey & Vincent, 2009).

However, the outcomes of inductions and deductions (i.e. the general conclusions of reasoning) and the examples and images provided in the text are also important on the concept image formation (Tall & Vincent, 1981). Additionally, while empirical reasoning in mathematics has an informal purpose in the scientific context it has a validating intention which leads to generating scientific knowledge. Having taken all the above issues into consideration we developed an interdisciplinary framework on reasoning in texts as we present in Chapter 3 and Chapter 4.

The role of visual-verbal relation

Adopting an activity theory perspective, visual representations are considered as 'elements' (i.e. the basic building blocks of activity) (Roth & Lee, 2007). The way these elements are used in the fields of science and mathematics and contribute to collective knowledge on periodicity is important for students' learning The importance of the multimodality approach has been argued by Kress & van Leeuwen (2006). They consider that visual and verbal elements in texts are two items of information which are interrelated as follows: by elaboration (when an item elaborates on the meaning of another by further specifying or describing it) and by extension (when an item extends the meaning of another by adding something new to it). In this study, the function of the visual representations in the reasoning process is investigated thoroughly.

Our main focus is on the ways visual tools function in textbooks in order not only to enable, mediate and shape thinking, but also to attain students' enculturation in the conceptual field of periodical phenomena in the different school subjects.

Chapter 3: METHODOLOGY

A grounded theory research approach (Strauss & Corbin, 1998) is adopted in this study. Our methodological framework is based on the qualitative inductive content analysis. Moreover, the technique of systemic networks (Bliss, Monk & Ogborn, 1983) has been adopted not only as a form of representing our scheme of categories, but also as an analytic tool. In particular, we aim to produce a quantitative elaboration of the arguments which underlie the text in 11 Greek textbooks on topics related to the notion of periodicity.

The textbooks sample

The texts analysed are taken from the subjects of Astronomy, Mathematics, Physics and applied technologies (Electrology, Electronics and Informatics) used in Greek lower secondary and upper secondary General and Vocational school. In each textbook we restrict our analysis to topics that are related to periodicity. Specifically in Mathematics the topics are trigonometry and periodic functions, in Physics the topics are related to Periodic phenomena (e.g. oscillations, simple harmonic and circular motion) while in applied technologies the topics are related mostly to Alternate Currents. We analyzed texts from 11 textbooks (the whole chapters and the proposed exercises or selected units).

Table 3.1

Grade/ School subjects	**Lower secondary school (GYMNASIO)** **Textbooks and selected chapters**		
	Physics	**Mathematics**	**Astronomy**
9	**Ch. 4: Oscillations** *http://digitalschool.minedu.gov.gr/modules/ebook/show.php/DSGYM-C201/368/2458,9396/unit=917*	**Part 2/Ch. 2: Trigonometry** *http://digitalschool.minedu.gov.gr/modules/units/?course=DSGYM-C104&id=382*	**-**
General Upper Secondary Schools (GENERAL LYKEIO)			
Grade and Direction	**Textbooks and selected chapters**		
11a (Common core subjects)	**Ch. 1.2: Circular Motion** http://digitalschool.minedu.gov.gr/modules/units/?course=DSGL-A103&id=1041 **Ch. 5: Mechanical Oscillations** *http://digitalschool.minedu.gov.gr/modules/ebook/show.php/DSGL-B128/110/866,3221/unit=2117*	**Ch. 3: Trigonometry** *http://digitalschool.minedu.gov.gr/modules/ebook/show.php/DSGL-B112/104/997,3601/unit=1634*	**Selected units** from the following Chapters: **Ch. 1**: Astronomical observations and tools & **Ch 2: The solar system** http://digitalschool.minedu.gov.gr/courses/DSGL-B114/
11b **(Positive and Technological Directions)**	**Selected units from Ch 3: Electrical field** (p. 102, 103) *http://digitalschool.minedu.gov.gr/modules/ebook/show.php/DSGL-B101/280/2007,6823/unit=1053*		

	Selected units from Ch. 5: 5.6;5.7;5.9;5.10: Electromagnetical Induction *http://digitalschool.minedu.gov.gr/modules/ebook/show.php/DSGL-B101/280/2007,6825/unit=1055*		
12a (Common core subjects)	-	-	-
12b **Positive and Technological Directions**	**Ch. 1: Electrical and Mechanical Oscillations** *http://digitalschool.minedu.gov.gr/modules/units/?course=DSGL-C108&id=1064*	Selected units **Ch 2.4: Trigonometric form of a complex number** *http://digitalschool.minedu.gov.gr/modules/units/?course=DSGL-C105&id=1026*	
12b **Applied Technologies (Electrology)**	**Selected units from Ch. 1: Electrical circuits (1.4-1.7)** ***http://digitalschool.minedu.gov.gr/modules/units/?course=DSGL-C123&id=1650***		

Table 3.2			
Technical Vocational Educational Schools (Professional Lykeio-EPAL)			
Specializations	**Electrology**	**Electronics**	**Informatics**
Grade 11 (B class EPAL)			
Subjects **Textbooks**	ELECTROTECHNICS Ch 5.1 Alternative Current (AC)	CIRCUITS Ch. 9 (AC)	NETWORKS Selected Unit 1.1.4

The texts (Units of analysis) from the selected textbooks

In order to implement our analytic plan, we divided the text into units of analysis by restricting analysis to all the parts which aim at delivering mathematical and scientific knowledge (we did not include working examples and historical notes). In this paper the word ‘text’ is used to denote a section of textbook material and the accompanying visual representations.

Our unit of analysis is every conceptual thematic unit that has an independence from the rest of the text and produces an argumentation. It is conceived as a part of the topic that we analyze; it has a beginning and an end; and has a relative independence in its content: we can identify it and distinguish it from the other units. Each unit of analysis is characterized by its thematic content (e.g. “Define periodic function” or “Define periodic motion” or “Describe the generation of alternate current”) which is organized in a particular way. One unit of analysis several times coincides with a textbook unit as it is defined by the author. But in some cases we have to split the textbook unit in more units of analysis when a change in its thematic content and the argumentation produced is identified.

The specific units analyzed in each subject and each grade are presented in Table 3.3.

Table 3.3: The units analyzed in each school subject		
Thematic content	Grade No	The specific topics
MATHEMATICS		
Trigonometry	Grade 9	1. Define trigonometric numbers from 0^o to 90^o by using the Pythagorean theorem (p. 232) 2. Defining trigonometric numbers from 0^o-180^o (p. 23)
Trigonometry	Grade 11a	1. Trigonometric numbers (0-90^o) as independent of the specific triangles quantities (p.49-50) 2. Trigonometric numbers (0-360^o) (p.50) 3. Trigonometric numbers $>360^o$ (p.51) 4. Relation (k360+ω & ω in degrees) (p.52) 5. Define the trigonometric circle (p.53) 6. Define the sine &cosine axis on trig. Circle (p.53-54) 7. Define the tangent axis on trigonometric Circle (p.54) 8. Define the rad (p.55) 9. Basic trigonometric identities on the basis of the trigonometric circle (p. 60) 10. Basic trigonometric identities on the basis of Mathematical relations (p. 61) 11. Define periodic functions (p.72-73) 12. The periodicity of all the trigonometric functions (p.74-75 13. Graphing the sine function (p.75-77) 14. Graphing the cosine function (p.78) 15. Graphing the tangent function (p.78) 16. The r, ω of f(x)=rsin(ωx) (p.81) 17. Solving the trig equation sinx=a (p.83) 18. Solving the trig equation cosx=a (p.85) 19. Solving the trig equation tan=a (p 86) 20. The equation f(x)=ρsin(x) (p.108-109) 21. Conversion from asinx+bcosx to rsin(x+φ) (p.109-110) 22. Parametric equations of the cycle
Trigonometric form of complex numbers	Grade 12b	1. Introduction 2. Complex argument 3. Trigonometric form **4.** Trigonometric form of the product of complex numbers **5.** De Moivre Theorem
PHYSICS		
Oscillations	Grade 9	1. Define periodic motions (general) (p.89) 2. Define oscillations (general) (p.89)

		3. The role of force in oscillation (p.90) 4. Relation of force-x in Simple Harmonic Oscillation (p. 90) 5. The main characteristics of oscillation & their relations (p.91-92) 6. Describe the simple pendulum (p.92) 7. Relation period (T) & other properties(mass of pendulum/length pendulum/geographical position in the case of pendulum/amplitude in the case of pendulum (p. 92) 8. Energy and oscillation (p.92-93)
Circular motion & Mechanical Oscillations	Grade 11a	1. Define simple circular motion & its main characteristics (SCM) (p.10-11) 2. Define linear velocity(υ) of simple circular motion (p.10-11) 3. Define angular velocity (ω) & its relation to other characteristics (p. 12) 4. Define the centripetal acceleration (p. 13) 5. What generates a circular motion (the role of centripetal force) (p. 15-16) 6. Define periodic phenomena & their main characteristics (p. 201-202) 7. Define different kind of periodic motions (p.202-203) 8. Linear harmonic oscillating (LHO)spring (p.204-205) 9. Amplitude of LHO (p.205) 10. The sinusoidal equations x(t); u(t); a(t) in LHO (p. 205-207) 11. Empirically the relation period T - mass (m) & the quality of spring (p.208-209) 12. Define the LHO and main characteristics (p.209-212) 13. Total Energy of an oscillation (p.210-211) 14. Simple pendulum as a Linear Harmonic Oscillator (p.213-214) 15. Relation of period (T) with the spring material - m (mass of an object)- mass of pendulum - length pendulum geographical amplitude& amplitude. (p.214-215)
	Grade 11b	1. Describe how the oscilloscope works (p.102) 2. How we can measure the alternate voltage (p.103) 3. Generate(rotating frame) and define alternate voltage (p.195-197) 4. Define alternate current (p.196-197) 5. How the generator of alternate Voltage functions (p.199-201)

Electrical and Mechanical Oscillations	Grade 12b	1. Define periodic phenomena and its properties (p. 8) 2. Kinematics of simple harmonic oscillations (p. 9-11) 3. Dynamics in SHO or mathematical relation of force-x in SHO (p.11-12) 4. Energy and SHO (p. 12-13) 5. Define Electrical oscillations (p.14) 6. Characteristics of Electrical oscillations (p. 15-16). 7. Simple Harmonic and electrical oscillation (p. 16) 8. Damped mechanical oscillation 9. Damped electrical oscillation 10. Forced mechanical oscillations (p.21) 11. Forced electrical oscillations (p.23) 12. Synthesis of oscillations 13. Synthesis of SHO f1=f2 14. Synthesis of SHP f1 $\neq$ f2
ASTRONOMY		
Solar system	Grade 11	1. Celestial sphere (p.21-22). 2. Define horizon (p.22-23) 3. The apparent motion of the sun from an observer from the earth & the three critical points in the sun' s daily path (p. 24). 4. Summer and Winter Solstice (p.24-25) 5. Explaining the apparent motion of the sun (p. 25) 6. The ecliptic (p. 25-26). 7. The apparent motion of the planets (p. 40-41). 8. The phases of the planets(p. 41). 9. The history of exploring the real motion of the sun (p.41-42). 10. Keppler 's laws (p.42-43). 11. The phases of the moon (p. 50). **12.** The tides (p. 51-52)
APPLIED TECHNOLOGIES		
GENERAL LYKEIO		
Electrical Circuits	Grade 12b	1. Alternating current (AC) 2. Generation of alternating voltange/current 3. Vector reprsentation_AC
TECHNICAL VOCATIONAL EDUCATIONAL SCHOOLS (EPAL)		
Electrotechnics	Grade 11	1. Changing currents(p. 332-333) 2. Periodic currents (p. 333-337) 3. Alternate current (p.338-341) 4. Generation of alternating current/ voltage (p.342-343) 5. Alternate currents and its characteristic properties

		(p. 344-345) 6. Alternating voltage and its characteristic properties (p. 344-345) 7. Vector representation of alternating functions (p. 350-351) 8. Alternate currents in phase (p. 352-354).
Circuits, continuous and alternate currents	Grade 11	1. Alternate current and its characteristics (p.260). 2. Alternate voltage and its characteristics (p.261). 3. Generation of alternate current (p. 262) 4. Rms voltage & current (p.263-264) 5. Vector representation of alternate elements (p. 264-265) 6. Basic circuits in alternate current (general) (p. 264-265) 7. Basic circuits in alternate current. Ohmic resistance in AC. (p. 266) 8. Basic circuits in alternate current. Spool in AC (p. 267) 9. Basic circuits in alternate current - Capacitor in AC (p. 268) 10. Complex circuits- RL in Series (p. 269) 11. Complex circuits- RC in Series (p. 270-271) 12. Complex circuits- RLC in Series (p.272) 13. Complex circuits- RLC in parallel (p.275-278) 14. Power of alternate current (p. 280-281)
Mathematical representation of a signal	Grade 11	1. Mathematical representation of a signal. 2. Signal as a harmonic oscillation.

The sample

In the Table 3.4 we present the whole sample analyzed (i.e. units of analysis; VRs; proposed exercises).

Table 3.4: The sample analyzed in the dimensions of textual units, VRs and proposed exercises

GENERAL SUBJECT	**SUBJECT GRADE_No**	**Textual units No**	**VRs No**	**Proposed exercises No**
MATHEMATICS	Math_Gr9 (**GYMNASIO**)	2	7	7
	Math_Gr11_Common Core (**GENERAL LYKEIO**)	22	38	59
	Math Gr12_Positive & Tech direction (**GENERAL LYKEIO**)	5	4	19
TOTAL MATHEMATICS		***29***	***49***	***85***
PHYSICS	Phys_Gr9 (**GYMNASIO**)	8	(11)	2
	Phys_Gr11_Common Core (**GENERAL LYKEIO**)	15	(33)	32
	Phys_Gr11 Positive & Tech direction (**GENERAL LYKEIO**)	5	(17)	-
	Phys_Gr12 Positive & Tech direction (**GENERAL LYKEIO**)	14	(40)	24
TOTAL PHYSICS		***42***	***101***	***58***
ASTRONOMY	Astronomy_Gr11_common Core (**GENERAL LYKEIO**)	12	23	-
ASTRONOMY		***12***	***23***	**-**
APPLIED TECHNOLOGIES	*Electrical Circuits _ Gr12* (**GENERAL LYKEIO**)	3	3	-
	Electrotechnics_Gr11 (EPAL)	8	18	
	Circuits_Gr11 (EPAL)	14	18	4
	Informatics_Gr11 (EPAL)	2	2	15
APPLIED TECHNOLOGIES		**27**	**41**	**19**
TOTAL		**110**	**214**	**162**

How each subject participate in the analysis

In Figures 3.1, 3.2, 3.3 we present in pie representations how each subject (mathematics, physics, astronomy and applied technologies participated in the three dimensions (i.e. units of analysis, VRs, proposed exercises.

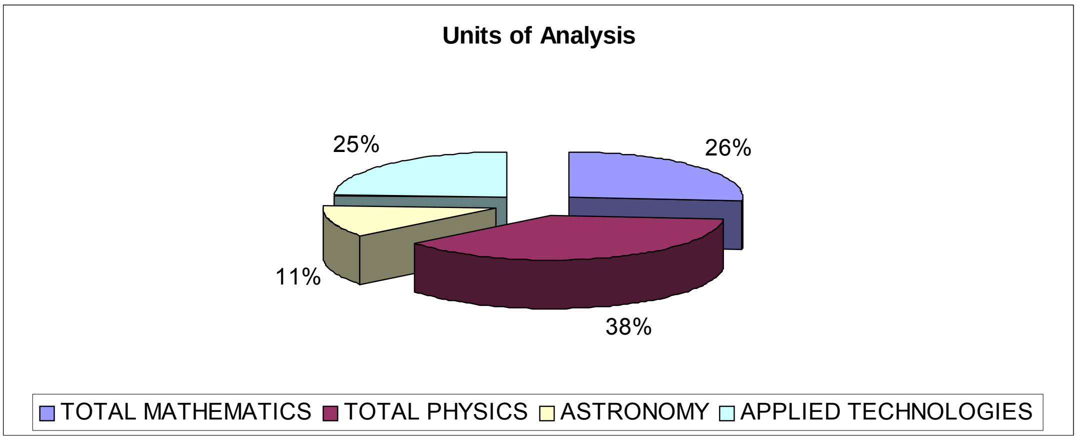


Figure 3.1: How each subject participates in the analysis. The case of the units of analysis.

Comments on Figure 3.1

The first in participation was the subject of Physics (42 units in 110 Units of analysis), the second was the subject of Mathematics (29 units), the third is the applied technologies (27 units) and the last subject was Astronomy (12 units).

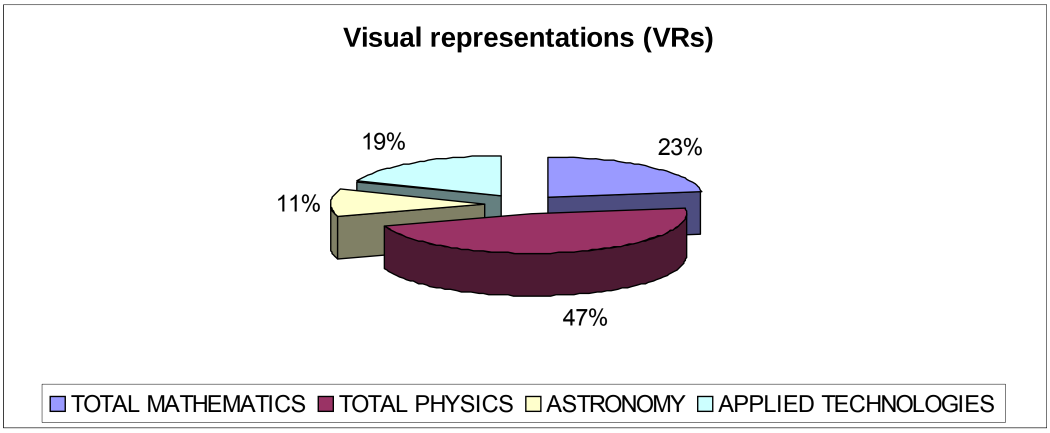


Figure 3.2: How each subject participates in the analysis. The case of VRs.

Comments on Figure 3.2

The subjects follow the same order as in Figure 1 but with slightly different proportions. Particularly, the first in participation was the subject of Physics (101 units & 214 VRs analyzed), the second was the subject of Mathematics (49 units), the third was that of applied technologies (41 units) and the last subject was Astronomy (23 units).

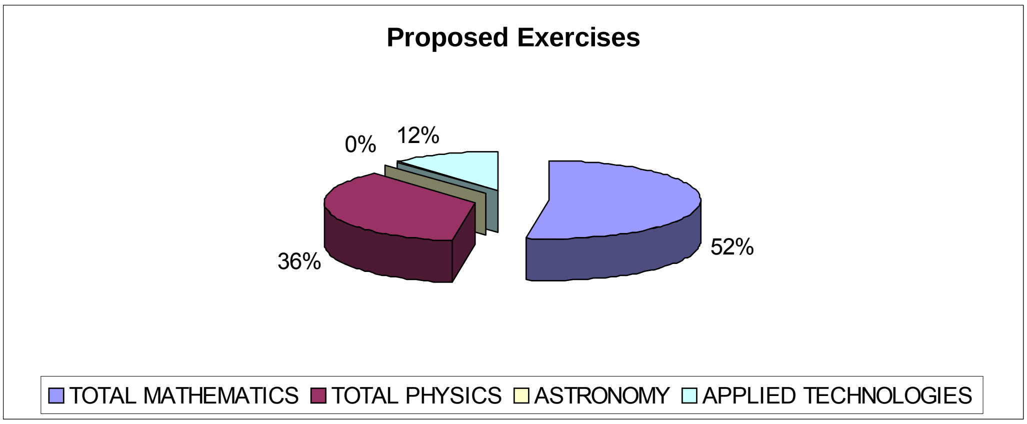


Figure 3.3: How each subject participates in the analysis. The case of the proposed exercises.

Comments on Figure 3.3

In the dimension of the proposed by the author exercises the first in participation was the subject of Mathematics (85 units in 162 exercises analyzed), the second was the subject of Physics (58 units), the third was applied technologies (19 units) while Astronomy was not participated in this dimension.

Qualitative/interpretative Data analysis

Qualitative inductive content analysis (Mayring, 2000) and the grounded theory research approach (Strauss & Corbin, 1998) has been employed for the analysis of all the texts in the dimension of the argumentation developed in all textual units, the VRs and the proposed exercises. Inductive approaches are intended to aid an understanding of meaning in complex data through the development of summary themes or categories from the raw data.

Initially, we define the conceptual thematic units that have an independence from the rest of the text, are characterized by one thematic content (e.g. "Define Linear Harmonic oscillation"), and produce an argumentation from the particular way they are organized. The process of argumentation in each text is generated as a sequence of interdependent and logically connected statements that lead to a claim in terms of the concept of periodicity aiming to persuade students for its truth. So, a secondary unit of analysis is defined by either one sentence or a sequence of sentences and the accompanying visual representations (VRs) that produce a type of reasoning and support the generation of argumentation developed in the thematic unit. Semantically, in each unit of analysis we can identify different explanations, justifications and/or proof of new knowledge. We call these types of reasoning as 'modes of reasoning' as this term is used in Stacey and Vincent's (2009) study.

Subsequently, we analyze the kind of modes of reasoning applied. The codes developed and negotiated among the researchers within a feedback loop. Those codes were revised and eventually reduced to main categories and checked in terms of their reliability. As a result, categories and subcategories were formed and their interrelations were recognised by matching our emerging classification to our data. Finally, the nature and the elements of argumentation were organised in the form of a systemic network (Bliss, et al. 1983).

Moreover, after having analyzed a lot of textual units, we focused on the role of VRs in the argumentation process. Through this study we identified the particular ways that VRs function in the different modes of reasoning.

The systematic qualitative content analysis of all the textual units (110 units) and the accompanying 214 Visual representations and 162 proposed exercises have led us to the production of the following schemes of categories:

A) The conceptual aspects of periodicity

B) The modes of reasoning as parts of the argumentation adopted by the authors in presented the new knowledge.

C) The VRs that accompany the modes of reasoning.

D) The visual verbal relation.

E) Finally, the analysis of 162 proposed exercises in the subjects of mathematics, physics and applied technologies has led us to the production of the following dimensions: a) demands; b) type of VRs; c) the context every exercise is placed.

The structure of the above category schemes is finalized after a number of reconstructions checking the categories through data.

After finishing the qualitative analysis on all the above dimensions we analyzed the final produced schemes in the form of systemic networks for each thematic unit in the selected texts and we quantified them by counting frequencies of appearance for each subject.

Finally, we shall present some additional issues that we spotted through our analysis that might affect students' understanding.

Chapter 4: ANALYSIS AND RESULTS

4.1) Conceptual field

We take the position that the inner nature of a concept of periodicity as a "system" (von Bertalanffy, 1968). is enhanced through connections between different aspects of the notion. Moreover, the aspects of periodicity are related to other peripheral notions (e.g. Hook's Law; similar triangles; the area of a triangle or similar triangles; Kepler's Law) which are not considered less important but in this study we have focused on the conceptual system of periodic phenomena and how these are presented and described in the textbooks across different subjects.

In this direction, we characterize each text according to the main conceptual aspects of periodicity that are presented and further supported.

4.1a) Results from Qualitative analysis

The first dimension we identified in the selected texts is related to whether the notion of periodicity appears explicitly or implicitly. The category of explicit reference to periodicity locates subcategories according to the types of periodic motions described in each text, the functions that model the above periodic motions, whether the reference is to damped oscillations (where we meet repeated but not periodic motions) and to the periodical characteristics or properties of periodic motions (i.e. the period T; the frequency f; the angular velocity ω; the definition of the radian measure in mathematical texts). All the above subcategories could be selected simultaneously.

Specifically, the categories are as follows:

- Explicit (the notion of periodicity appears explicitly)
    - Types of periodic motions
        - General types (expressed mostly in a descriptive way) (GT)
        - Specific types (their physical models or what generates these motions (e.g. the trigonometric circle or the rotational frame)
            - Rotational (Circular/ uniform circular /elliptical) (Rot)
            - Mechanical Oscillations (Osc)
            - Electrical oscillations (Elec)
        - Functions modelling periodic motions
            - Sinusoidal or harmonic functions (Sin)
            - Sum of sinusoidal functions (Sum sin)
            - Other types of periodic functions (Oth_P)
    - Damped oscillations (Dam)
    - Characteristics of periodic motions (T, f, ω, rad) (Char)
- Implicit (hidden or not explicitly stated as periodic motions) types

In Figure 4.1 the systemic network presents the scheme of the categories of the conceptual aspects of periodicity. The BAR ([) notation signifies that all the categories are mutually exclusive, whereas the BRA ({) notation signifies that any number or even all of the categories can be selected simultaneously.

All the subcategories of types of periodic motions, or periodic functions are mutually exclusive except the case of electrical oscillation that could be simultaneously related to the rotational motion.

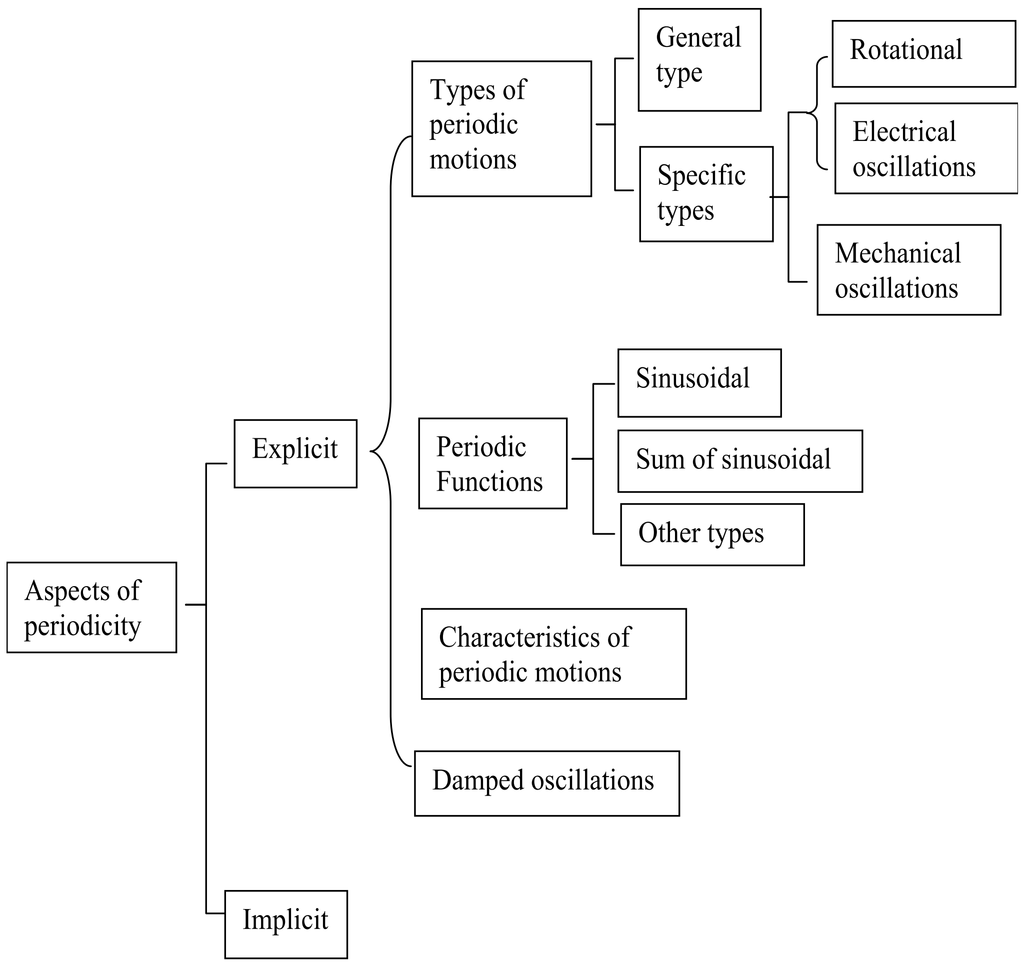


Figure 4.1: The systemic network on the aspects of periodicity in the school texts.

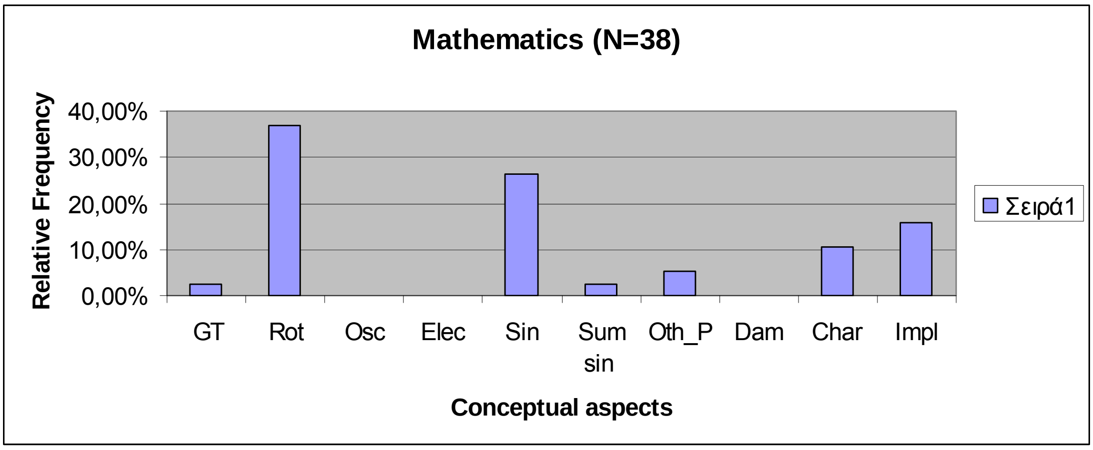
4.1b) Results from our Quantitative analysis

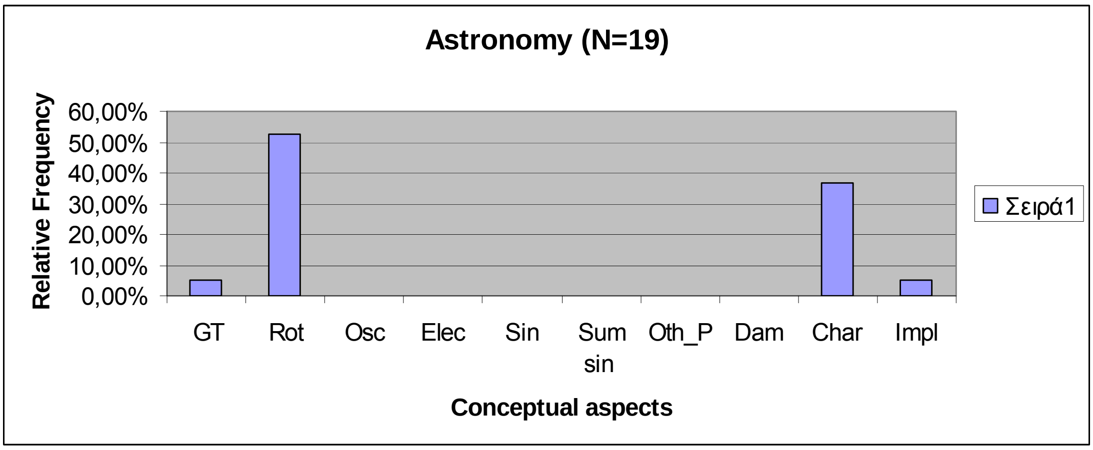
All thematic units (No=101) have been analyzed and the frequencies of categories' appearance have been counted. The final produced schema in the form of systemic network, for the dimension "conceptual aspects" of periodicity, has guided this type of analysis and counting. The results are presented in Table 4.1

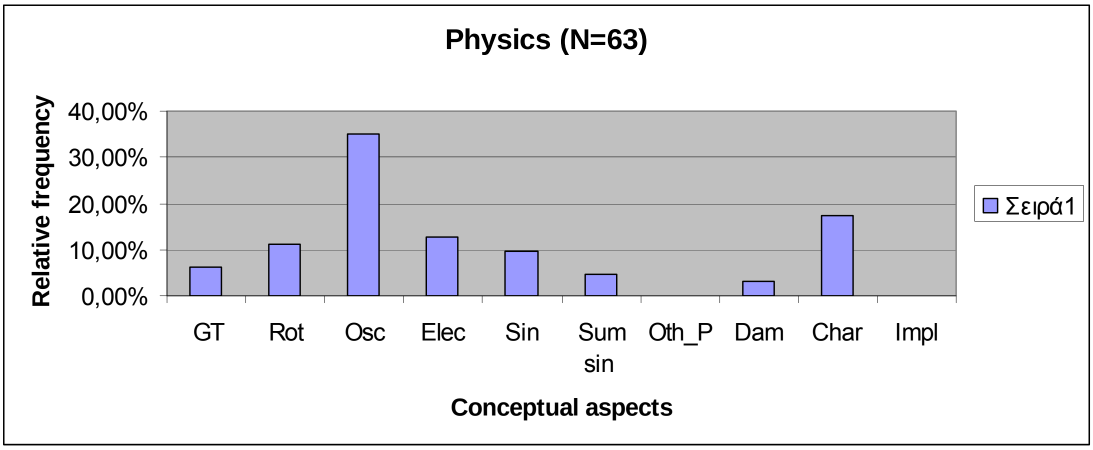
Table 4.1: The sample analyzed in all subjects and in different grades							
Conceptual aspects of periodicity				Mathematics N=29	Astronomy N=12	Physics N=42	Applied Technologies N=27
Explicit	Types of periodic motions	General type		1	1	4	4
		Specific type	Rotational	14	10	7	8
			Mechanical oscillation			22	
			Electrical oscillations			8	19
	Periodic functions	Sinusoidal		10		6	17
		Sum of sinusoidal		1		3	1
		Other types		2			
	Characteristics			4	7	11	5
	Damped oscillations					2	
Implicit				6	1		2

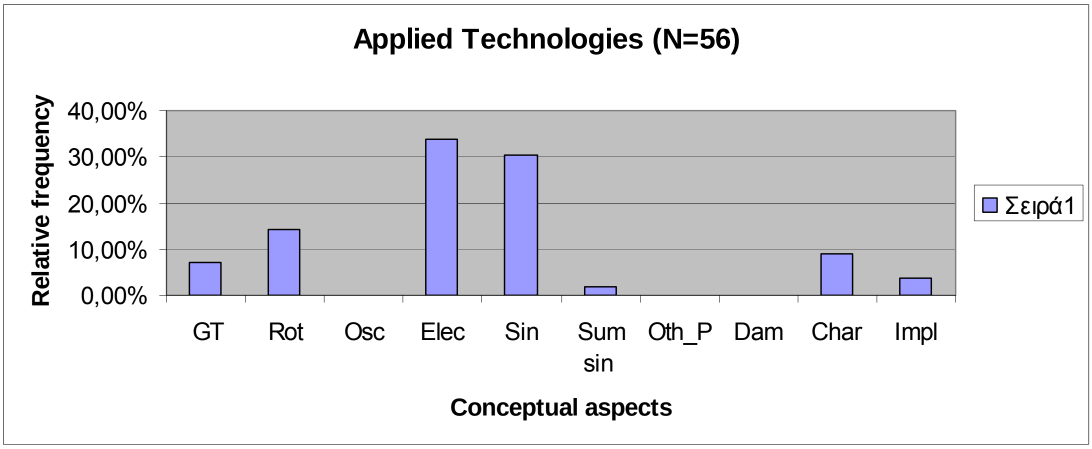
In Table 4.2 we present the percentage distribution of the conceptual aspects of periodicity identified in each subject. For example, the number of all the conceptual aspects identified in Mathematics texts are 38 while the case of Rotational periodic motions are represented the 14/38=37%.

Table 4.2: The Figural representations of Table 4.2 data in the different subjects









Issues emerged from Table 4.1 and 4.2

Physics is the most 'rich' subject where different aspects of periodicity are introduced explicitly and developed. On the other side, Astronomy is restricted to specific aspects of periodicity.

The most popular periodic motion in all subjects is the Rotational periodic motion. This type of periodic motion is modelled by the trigonometric circle in mathematics while the rotating frame is the basic model that generates the electrical current.

The most popular function that models the periodic behaviour of almost all periodic motions is the sinusoidal function. Only in the subject of astronomy the students do not meet this function. Moreover, in the texts of applied technologies sinusoidal functions are sharing a bigger part than in the mathematics.

The category of periodical characteristics also appears in all subjects.

In the case of General type of periodic motions we refer to periodic motions not explicitly, stated as rotational etc. The texts in the textbook of all subjects present this case mostly in the introductory texts.

The Mechanical oscillations as a subcategory of specific types of periodic motions are identified only in physics while Electrical oscillations are met in physics and applied technology texts.

The sum of sinusoidal functions (e.g. asinx+bcosx) as a subcategory of periodic functions is slightly referred to in school texts.

In mathematics texts a very good proportion (almost 15%) of all the conceptual aspects identified (N=38) in the selected units, the periodicity as a motion appears implicitly (e.g. in the thematic units of trigonometric numbers and trigonometric identities the periodic variation of the angle measure in relation to sine, cosine and tangent is almost hidden).

An important cognitive issue that emerged from our analysis is that the case of damped oscillations (in physics) appears as a case of periodic motions and not as a case of repeated but not periodic motions.

4.1c) Results from our Quantitative analysis: The case of conceptual field in Grade 11 mathematics and Physics texts

The only grade where all the students in the General and in the professional Lykeio meet and develop thoroughly and explicitly the notion of periodicity is Grade 11.

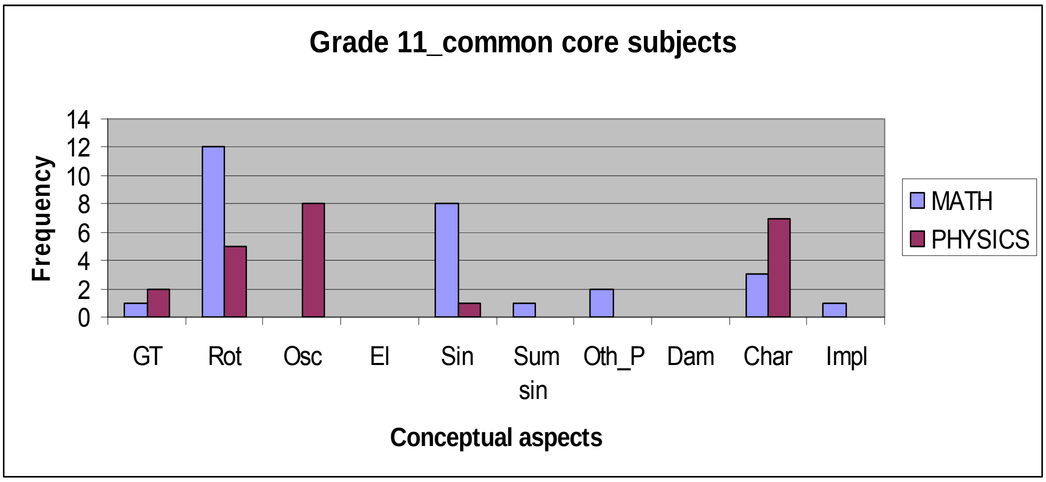


Figure 4.2: The frequencies in the Grade 11 mathematics and physics texts in relation to contextual meaning.

So, our research question is: what type of periodical aspects addressed to a grade 11 student in his common core subjects of mathematics and physics? Moreover, we analyze the data of these specific subjects because both are compulsory courses for any student in this grade in the General Education.

We present the frequency of the above conceptual aspects that meets the same student in his grade 11 class in General Education in Figure 4.2.

Issues emerging from Figure 4.2

Grade 11 in General Education is the level where every student meets and develops the notion of periodicity in a theoretical way.

It seems that physics is the school subject that provides mostly opportunities for the student to recognize and become familiar with the critical aspects of the notion. These results could support other research findings that claim that students understand periodicity as a process, while their concept image of periodicity is based on time dependent dynamic examples (Shama, 1998).

The sinusoidal function is mostly met in mathematics texts and not in physics texts.

Physics texts establish the characteristics of periodic functions in Grade 11.

In general we could conclude that the two subjects have a complementary function in terms of the conceptual system of periodicity. On the other hand, the role of the mathematics and science teacher is essential in order to integrate and incorporate the different aspects of periodicity addressed in the above subjects and support their students' conceptualization.

4.2) The argumentation developed in each text

By adopting the position that the inner nature of the concept of periodicity is enhanced through the argumentation developed in all texts, we analyzed the structure of the argumentation in all textual units. The structure of argumentation in each thematic unit is characterized in terms of its parts. The parts of the argumentation were acknowledged in terms of the kind of modes of reasoning applied by the author when developing the new knowledge.

The Modes of Reasoning are categorized as follows:

- *Nomo-logical*
  - when the reasoning of the text is based on axioms or theories or previously established statements and are the basis for further reasoning **(N1)** (e.g. *"Since the function f(x)=sinx is periodic with period 2π it is sufficient to study it in an interval that has length 2π, e.g. [0, 2π])*
  - when a definition, a generalization or a law emerges as a result of previous generalizations **(N2)** (*it is usually recorded in the text in a distinctive way e.g. in bold letters)*
  - when taxonomic definitions are met **(N3)** (e.g. *the simple harmonic oscillations are a particular type of mechanical oscillations which are a particular type of periodic motions***)**
  - When historic data are presented **(N4)**
- *Logical-Mathematical when they are based on:*
  - Applying mathematical relations and techniques **(LM)** (e.g. sketch a graph or apply and transform algebraic expressions). (See an example on Figure in Figure 4.3).

$$F = ma \qquad (1.5)$$

Η (1.5) γίνεται από την (1.3)

$$F = -ma_{max}\,\eta\mu\omega t \quad ή \quad F = -m\omega^2 A\eta\mu\omega t \qquad (1.6)$$

και επειδή $x = A\eta\mu\omega t$ η (1.6) γίνεται

$$F = -m\omega^2 x \qquad (1.7)$$

Figure 4.3: An example of LM mode of reasoning.

Physics, Grade 12b.

- *Logical – Empirical*, when experiences are either related to logical conclusions or linked general statements with examples and specific situations. These reasoning appears to have a peculiarity and can be further discerned in the following categories:
  - *Application reasoning* that starts from a general idea of logical type and ends up in implementing it in certain empirical or specific situations **(LE1).** (e.g. **"***The normal circular movement of the Sun is periodical as well as the movement of the Earth round the sun, which is repeated annually")*
  - Reasoning that starts from specific situations or empirical data and ends up in general phrasings, meanings or conclusions **(LE2).**

- If the data are based on scientific experimental representations **(LE2a)**
- If the data are based on geometrical representations and mathematical models **(LE2b)**
- If the data are based on representations related to the science of astronomy **(LE2c)**

o *Explanatory reasoning* which aims to explain theoretical ideas or exploit invented situations to explain phenomena. **(LE3**). (See an example of LE3 reasoning in Figure 4.4).

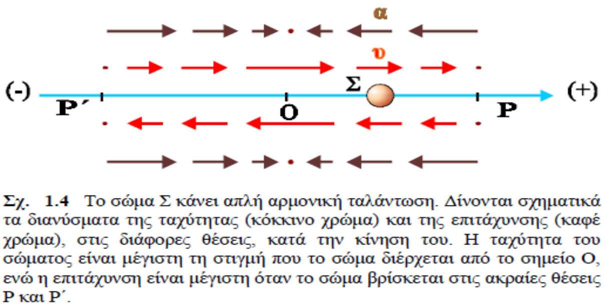


Figure 4.4: An example of LE3 mode of reasoning.

Physics, Grade 12b.

- *Empirical reasoning*
  - o *Recalling experiences* from everyday life **(E1)**
  - o *Describing enactive experiences* either of everyday life or in an experimental activity **(E2).**

In Figure 4.5 we present the systemic network we produce on the dimension of modes of reasoning as parts of the argumentation developed in all texts.

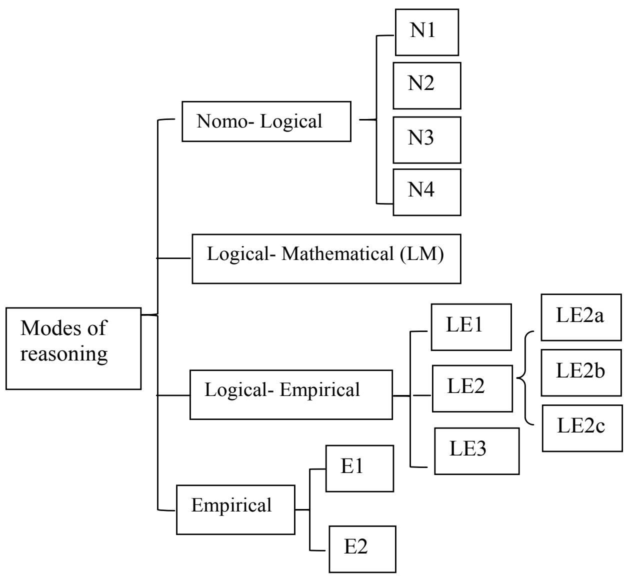


Figure 4.5: The systemic network on the dimension of modes of reasoning

4.2a) Qualitative analysis across subjects

In the section 4.2a we exemplify our analysis in two texts from the subjects of physics and mathematics that share a closely related thematic content and we highlight issues that emerge from our analysis. In section 4.2b we present the results from our quantitative analysis and the issues emerging.

Example 1: Physics, Grade 9.

The text is from the topic 'Oscillations' and its thematic content is: "Define periodic motions".

E1 *"When you were younger you would have got into a swing many times or you would have even noticed the other kids playing with it.*

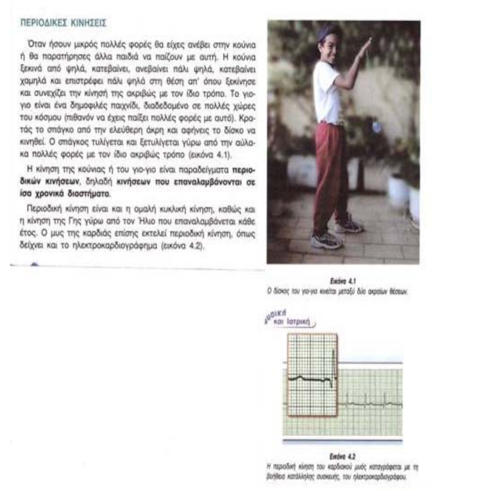
E2 *The swing has a high starting point, goes up and down and back to its starting point and keeps on moving in the exact same way.*

E1 *The yo-yo is a popular game, widely used in many countries in the world (maybe you have played with it several times).*

E2 *You hold the string from the one edge and you let the circle move. The string winds and unwinds around the spinning axle several times in exactly the same way.*

N2 *The movements of the swing or the yo-yo are examples of* ***periodic motions. This means that they are motions that are repeated at equal intervals.***

LE1 *The normal circular movement of the Sun is periodical as well as the movement of the Earth round the sun, which is repeated annually. The muscle of the heart performs a periodic motion as presented at the electrocardiogram"*



ΠΕΡΙΟΔΙΚΕΣ ΚΙΝΗΣΕΙΣ

Όταν ήσουν μικρός πολλές φορές θα είχες ανέβει στην κούνια ή θα παρατήρησες άλλα παιδιά να παίζουν με αυτή. Η κούνια ξεκινά από ψηλά, κατεβαίνει, ανεβαίνει πάλι ψηλά, κατεβαίνει χαμηλά και επιστρέφει πάλι ψηλά στη θέση απ' όπου ξεκίνησε και συνεχίζει την κίνησή της ακριβώς με τον ίδιο τρόπο. Το γιο-γιο είναι ένα δημοφιλές παιχνίδι, διαδεδομένο σε πολλές χώρες του κόσμου (πιθανόν να έχεις παίξει πολλές φορές με αυτό). Κρατάς το σπάγκο από την ελεύθερη άκρη και αφήνεις το δίσκο να κινηθεί. Ο σπάγκος τυλίγεται και ξετυλίγεται γύρω από την αύλακα πολλές φορές με τον ίδιο ακριβώς τρόπο (εικόνα 4.1).

Η κίνηση της κούνιας ή του γιο-γιο είναι παραδείγματα **περιοδικών κινήσεων**, δηλαδή **κινήσεων που επαναλαμβάνονται σε ίσα χρονικά διαστήματα**.

Περιοδική κίνηση είναι και η ομαλή κυκλική κίνηση, καθώς και η κίνηση της Γης γύρω από τον Ήλιο που επαναλαμβάνεται κάθε έτος. Ο μυς της καρδιάς επίσης εκτελεί περιοδική κίνηση, όπως δείχνει και το ηλεκτροκαρδιογράφημα (εικόνα 4.2).

Figure 4.6: Physics Grade 9. The text

(Physics, 3rd Grade of Lower Secondary School. (2008). Athens, Greece: OEDB, p. 89)

We can see how the development of argumentation in this thematic unit is produced. The sequence of a number of empirical modes of reasoning, a nomo-logical mode and a logical-empirical mode are the units/parts of the argumentation that aims to present students the periodic motions and to persuade them for their characteristics and properties existed in a number of real life situations.

Example 2: Mathematics, Grade 11 Common Core Subject.

The text is from the topic of Trigonometry. Its thematic content is: "Define a periodic function and its period".

LE2b *"Suppose that a ferry travels between two ports, A and B, and the graphic representation of its distance from port A as a function of time is presented in the following graph* [Figure 4.7a]. *We notice that every 1 and 1/2 hour the* *ferry repeats the exact same movement. This means that in whatever distance it is from port A in some time (t) it will be at the same distance at the time (t+1½) hours and it was at the same distance on the (t-1½) hours. Consequently, the function that presents the distance of the ferry from port A, in respect to the variable t takes the same values at t, t + 1½ and t-1½ .We suggest that this function is periodic with a period of 1 ½ hours.*

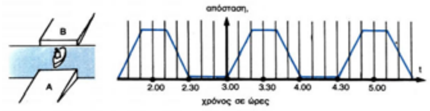


Figure 4.7 a

LE2b

The following graph [Figure 4.7b] *is a graphic representation of the height of the swing as a function of time (t). We notice that despite the height of the swing in a certain moment (t), it will have the same height at the time (t+2)s as well as at (t-2)s. We say that the function (that models the height of the swing with respect to t) is periodic with a 2 sec period.*

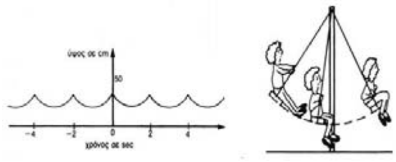


Figure 4.7b

In general:

N2

A function f with domain the set A is called periodic, when there is a real number T > 0 so as for every x ∈ A: i) x + T ∈ A, x - T ∈ A and ii) f(x + T) = f(x - T) = f(x). The real number T is called the period of f."

(Algebra, 2rd Grade of Upper Secondary School. (2012). Athens, Greece: OEDB, p. 73)

In this case, the argumentation is based on two different modes of logical-empirical reasoning and ends with a nomo-logical mode with main aim students' conceptualization of the abstract description of a periodic phenomenon, that of periodic function.

Issues emerging by comparing the argumentation developed in the two texts

By comparing and contrasting the argumentation in the two texts that share a closely related thematic content we get some evidence of how the contextual activity via reasoning is shaped in different subjects and in different grade levels.

Particularly, through our analysis, we spotted differences in the argumentation produced that could illuminate aspects of the notion in different ways.

Physics text starts with empirical modes of reasoning (E1 & E2) while mathematical text starts with Logical empirical (LE2) modes of reasoning. The text in physics bases its reasoning on students' experiences while the text in mathematics bases its reasoning on mathematical objects (the periodic graphs). Moreover, although a common example was used in both texts (the periodic motion of a swing) different modes of reasoning were employed that could support alternative operations on producing the inner nature of the notion.

The physics text concludes with logical-empirical LE1 (explanatory mode of reasoning) when starting from a general idea of logical type (the definition) and ending up by implementing it in certain empirical situations. On the other side, this mode of reasoning is absent in the mathematics text. The logical-empirical mode of reasoning LE1 is very common in physics that aims to provide more examples in order to reinforce students' understanding.

The common Nomo-logical mode of reasoning (N2) the definition of periodic motions in the science text comes as a generalization of verbalized properties while the definition of periodic functions in the mathematical text comes as a generalization of mathematical and symbolic properties. Moreover, the different definitions support different perspectives of the notion. Particularly, according to Van Dormolen and Zaslavsky (2003) the science text supports a holistic perspective while the mathematics text a point-wise one.

Some of these differences could easily be explained due to the difference in readers' school level (different school grades), while some others characterize the context in which each argumentation is developed.

Comparing and connecting the above alternative procedures could support students' conceptualizing activity.

4.2b) Quantitative analysis

We analyzed 95% of all the modes of reasoning in all texts qualitatively. The results of this analysis are presented in Table 4.3 while Figure 4.8 presents the same results in a Histogram representation.

Table 4.3: The relative frequencies on the dimension of modes of reasoning

Modes of reasoning			Mathematics N=29 %	Astronomy N=12 %	Physics N=42 %	Applied technologies N=27 %
Nomo-logical	N1		58.6	16.7	52.4	11.1
	N2		82.8	91.7	97.6	81.5
	N3		0	08.3	9.5	11.1
	N4		34	16.7	0.0	0.0
Logical-mathematical			48.3	0.0	35.7	22.2
Logical empirical	LE1		31.0	08.3	42.9	14.8
	LE2	LE2a	3.4	0.0	40.5	7.4
		Le2b	58.6	16.7	38.1	25.9
		LE2c	0	41.7	0.0	0.0
	LE3		0	33.3	14.3	0.0
Empirical	E1		0	41.7	9.5	0.0
	E2		20.7	41.7	38.1	48.1

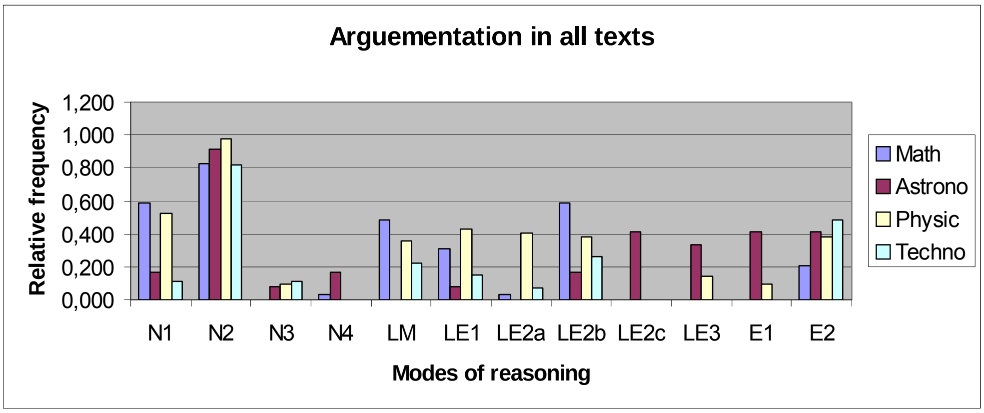


Figure 4.8: The figural representation from Table 4.3 data.

Issues emerged from Table 4.3 and Figure 4.8

Although all subjects contribute, in almost similar ways, to the above main categories of modes of reasoning (i.e. they all have nomo-logical, logical empirical and empirical modes of reasoning) we can make some interesting observations: The N3 (taxonomic category) is completely absent in mathematics. This may cause a problem in students' knowledge since they could not categorize different types of periodic motions or the functions that model their behaviour.

All texts incorporate a nomo-logical mode of reasoning in the form of N2 (a definition, or a generalization).

The historical aspects are very rear in all subjects (only in astronomy contribute to the argumentation developed).

As it was expected the category of Logical-mathematical is mostly present in mathematics but also in physics.

Application reasoning LE1 is a very common mode of reasoning in physics and less common in mathematics while in the other subjects its use is rather limited.

All the subjects use the Logical-empirical mode of reasoning in the category of LE2. In physics it is based on experimental demonstrations (LE2a) and in mathematical representations (e.g. vectors, or functions that model the periodic motions) (LE2b) while in mathematics it is based only on mathematical representations (LE2b).

The Logical-empirical mode of reasoning in the category of LE3 (defined as the case of explanatory reasoning that aims to explain theoretical ideas) is mostly met in astronomy and less in physics while it is absent in the other two subjects. Usually this mode of reasoning is accompanied by a visual representation. In Table 4.4 we present an example of this mode of reasoning in the subject of Astronomy.

Table 4.4: An example of LE3 mode of reasoning Ch 3: Astronomy. The solar system. Thematic unit: The phases of the planets (p. 41).	
Original text and accompanying VR	Translation
The VERBAL COMPONENT OF THE TEXT	
Στο σχήμα 3.10 παρουσιάζονται οι διάφορες φάσεις της Αφροδίτης κατά την περιφορά της γύρω από τον Ήλιο καθώς και η εξήγησή τους	In Figure 3.10 presented the different phases of Venus while it orbits the Sun and their explanations.
The VISUAL COMPONENT OF THE TEXT	
 *Σχήμα 3.10:* Η εξήγηση των φάσεων της Αφροδίτης: Ο παρατη-ρητής ανάλογα με τη θέση του ως προς την Αφροδίτη βλέπει ή ολόκληρο το φωτιζόμενο δίσκο της ή ένα μέρος του ή δεν το βλέ-πει καθόλου.	Scheme 3.10: The explanation of the phases of Venus. The observer sees parts of the luminous disk of Venus, the whole or none according to its position in relation to Venus.

Empirical mode of reasoning of type E1 (recalling every day experiences) is present only in the subjects of astronomy and physics. The other subjects never reason on the basis of students' every day experiences.

Finally, empirical mode of reasoning of E2 type (describing enactive experiences) is present in all subjects.

4.2c) Quantitative analysis of the subject of Physics across Grades

We will see how the modes of reasoning are changing through grades. In this case we take the subject of physics (Grade 9, 11 and 12). In Table 4.4. and Figure 4.10 we present the results of our analysis.

Table 4.4: Relative frequencies on the dimension of modes of reasoning: The case of physics across subjects					
PHYSICS					
Modes of reasoning			Grade 9 N=8 (%)	Grade 11 N=20 (%)	Grade 12 N=14 (%)
Nomo-logical	N1		37.5	50	64
	N2		100	100	93
	N3		0	10	14
	N4		0	0	0
Logical-mathematical	LM		25	30	0,50
Logical-Empirical	LE1		75	45	21
	LE2	LE2a	50	35	43

		LE2b	50	55	7
		LE2c	0	0	0
	LE3		0	10	29
Empirical	E1		25	10	0
	E2		50	45	21

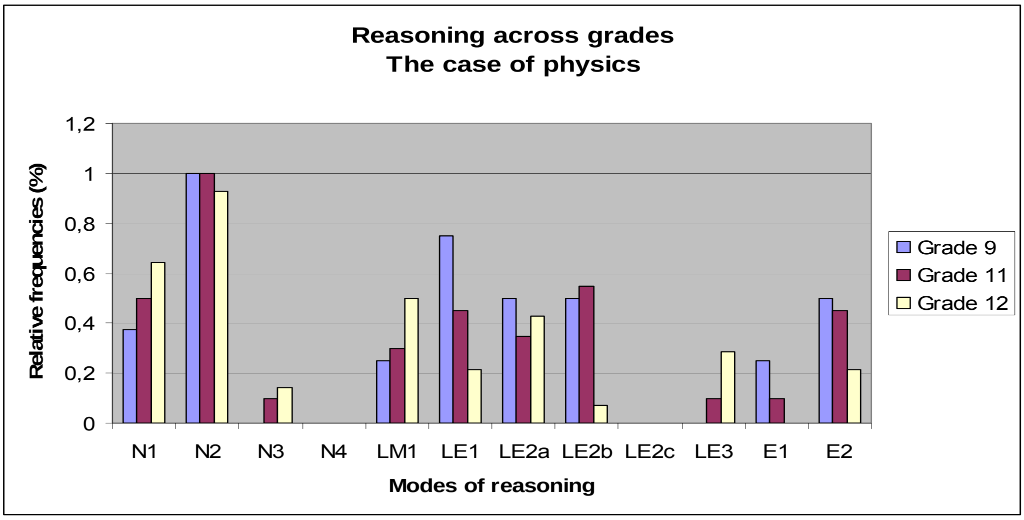


Figure 4.9: Figural representation of the Table 4.4 data

Issues emerged from Table 4.4 or Figure 4.9

The use of reasoning based on Nomo-logical N1 and Logical- Mathematical modes of reasoning is increased as we move from grade 9 to grade 12. On the other side, the use of explanatory reasoning LE1 and Empirical reasoning E2 seems to decrease.

Empirical reasoning E1 is absent in Grade 12.

The number of Logical- empirical modes of reasoning LE3 is increased from grade 11 to grade 12. The authors use this type of reasoning in physics texts in order to explain complex theoretical issues. The modes of reasoning that their presence do not change across grades are Nomo-logical N2 (definitions) and Logical-empirical LE2a.

Finally, LE2b that is based on specific situations on mathematical representations increases from Grade 9 to grade 11 and seems to disappear on Grade 12.

Logical-empirical and empirical modes of reasoning keep students close to the context and the notion, while Logical-mathematical modes of reasoning emerge in context free activities.

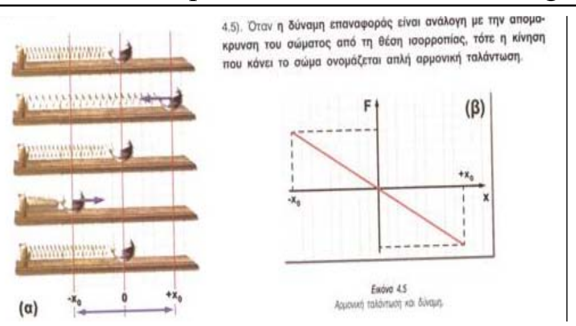


Figure 4.10: An example of LE2a and LE2b mode of reasoning. Physics, Grade 9.

For example, the way that the authors’ reason on the relation F-x in the case of Simple Harmonic oscillation in Grade 9 & in Grade

12 is characteristic. In Grade 12, the mode of reasoning is Logical- mathematical, as Figure 4.3 indicates, where the students have to transform algebraic expressions as follows: $F=ma= - masin(\omega t)=\ldots= - m\omega^2x$.

In Grade 9, the student must uncover rules and conventions on mathematical elements as Figure 4.10 shows.

The role of context is crucial in students' conceptualization and understanding. Moreover, educating students to reason, while taking into consideration the contextual elements, could contribute to their development of active monitoring and self-regulating competences (Kaiser, 2009).

4.3) Visual Representations (VRs)

Through our study, we have noticed that understanding periodicity demands connections between on the one hand the periodic phenomena of everyday life and natural world and on the other hand the abstract mathematical notions which model them. So we considered that such a dimension could be useful to be investigated. Also, the genre of the visual representation was considered as an interesting dimension and was used as a dimension of analysis in all texts.

4.3a) Qualitative analysis

The VRs are analyzed according to two dimensions: the *context* posed by the VRs and their *genre* by concentrating our interest on the features of periodicity presented.

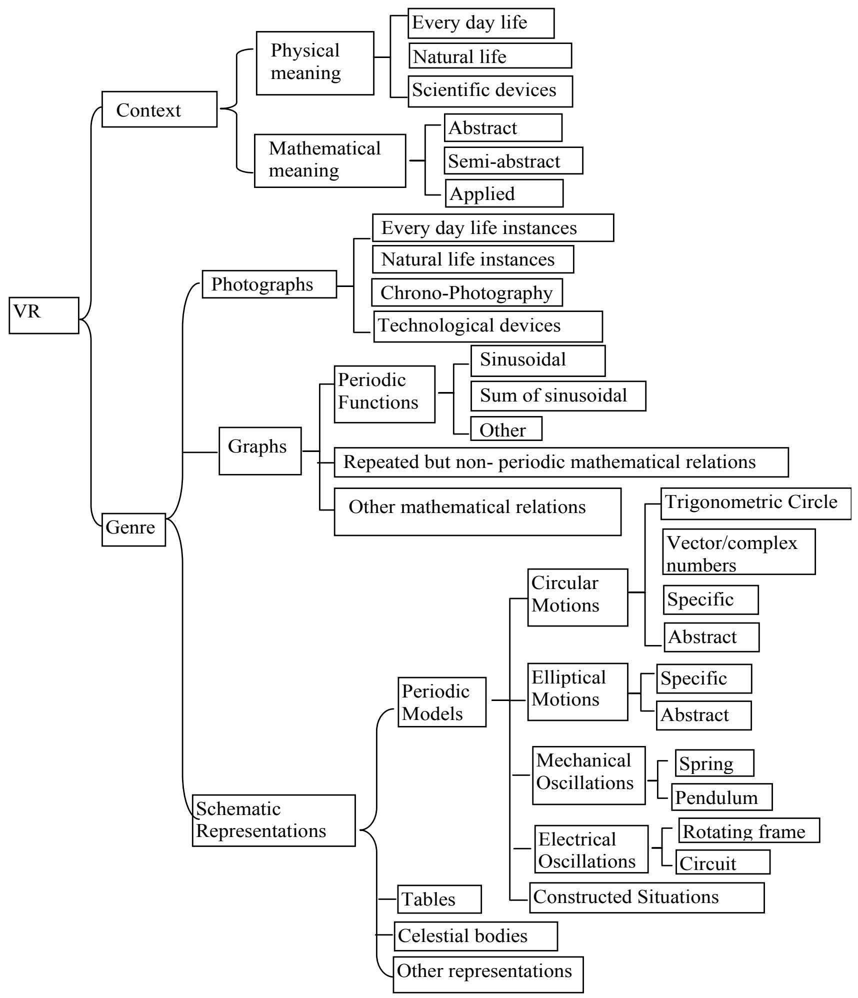


Figure 4.11: The systemic network. The case of Visual representations.

In terms of context, two further categories are defined: VRs with *physical meaning*, where the periodic phenomenon is placed in a context of everyday life, or natural life, or as a scientific device, and VRs with *mathematical meaning,* where the periodic phenomenon is presented in an abstract, or in a semi-abstract (e.g. graphs representing time-dependent processes), or in a specific context (e.g. numerical value charts).

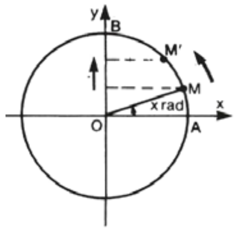
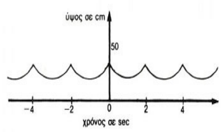
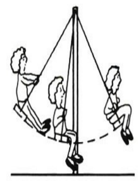
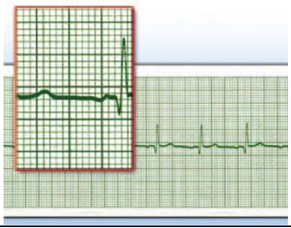
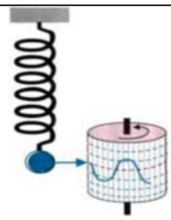
In terms of the genre of the VRs, three main categories are defined: the *photographs*, the *graphs* and the *schematic representations*. Although these categories are content free, the sub-categories are related to the features of their content.

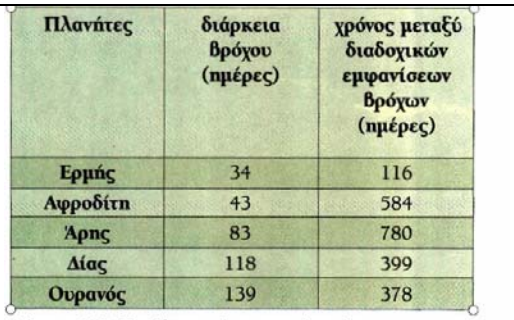
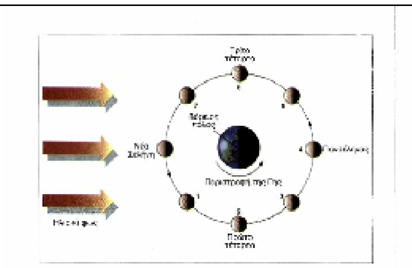
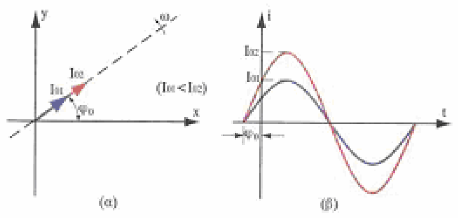
So, photographs present instances of (a) every day life examples and (b) natural world; or (c) chrono-photography of phenomena and (d) technological devices.

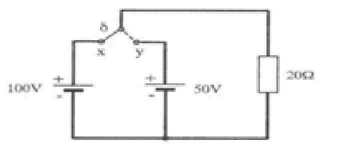
Graphs represent a) sinusoidal; (b) sum of sinusoidal functions; (c) repeated but non-periodic functions and (d) other mathematical relations.

Under the sub-category 'Schematic representations' fall representations of periodic models, tables or other VRs not explicitly related to periodicity (e.g. triangles). Periodic models may be circular; oscillatory (i.e. representations of spring or pendulum); electrical (VRs of circuits or rotating frames); or constructed situations. The trigonometric circle, vector representations of complex number, abstract or specific representations of circular periodic motions are used as models of this motion.

The systemic network in VRs is presented in Figure 4.11, while in Table 4.5 we exemplify our analysis in VRs from the subject of Mathematics, Science, Astronomy and Applied Technologies.

Table 4.5: Examples of VRs analysis	
	 
Mathematics Grade 11 *Context* Physical meaning: - Mathematical meaning: Abstract *Genre* Schematic representation: periodic model Circular motion Trigonometric circle	Mathematics Grade 11 *Context* Physical meaning: Every day life Mathematical meaning: semi-applied *Genre* Schematic representation/ Graph/ Periodic function/Other & Schematic representation/periodic model/constructed situation
	
Physics Grade 9 *Context* Physical meaning: Every day life Mathematical meaning: Applied	Physics Grade 11 *Context* Physical meaning: scientific device Mathematical meaning: semi-abstract

<table>
<tr>
<td>Genre
Photo/ Every day life instances</td>
<td>Genre
Schematic representation/
Periodic model/
constructed situation</td>
</tr>
<tr>
<td>
<table>
<tr><th>Πλανήτες</th><th>διάρκεια βρόχου (ημέρες)</th><th>χρόνος μεταξύ διαδοχικών εμφανίσεων βρόχων (ημέρες)</th></tr>
<tr><td>Ερμής</td><td>34</td><td>116</td></tr>
<tr><td>Αφροδίτη</td><td>43</td><td>584</td></tr>
<tr><td>Άρης</td><td>83</td><td>780</td></tr>
<tr><td>Δίας</td><td>118</td><td>399</td></tr>
<tr><td>Ουρανός</td><td>139</td><td>378</td></tr>
</table>
Πίνακας 3.7: Η ανάδρομη κίνηση των πλανητών.</td>
<td>
Σχήμα 3.22: Οι φάσεις της Σελήνης. Ανάλογα με τη σχετική θέση</td>
</tr>
<tr>
<td>Astronomy Grade 11
Context
Physical meaning: Natural life
Mathematical meaning: Applied
Genre
Schematic representation: Table</td>
<td>Astronomy Grade 11
Context
Physical meaning: Natural life
Mathematical meaning: Applied
Genre
Schematic representation/circular motion/specific</td>
</tr>
<tr>
<td>

Σχήμα 5.1.18. Διανυσματική και χρονική παράσταση ημιτονικών ρευμάτων</td>
<td>

Σχήμα 5.1.2. Κύκλωμα παραγωγής περιοδικού ρεύματος</td>
</tr>
<tr>
<td>Applied technologies Grade 11
Context
Physical meaning: -
Mathematical meaning: Applied
Genre
Schematic representation: Periodic model/circular motion/complex numbers
&
Graph/periodic function/sinusoidal</td>
<td>Applied technologies Grade 11
Context
Physical meaning: scientific device
Mathematical meaning: Applied
Genre
Schematic representation: Periodic model/electrical oscillations/ circuit</td>
</tr>
</table>

4.3b) Quantitative analysis: The category of VRs contextual meaning

Counting of relative frequencies of appearance of the final produced schemes in the form of systemic network for the dimension of context on all the VRs analyzed (No=214) in each subject gave us Table 4.6, while Figure 4.12 represents the same results in a visual form.

The categories of physical and mathematical meaning can exist simultaneously. So, any VR could have either a physical and/or mathematical meaning.

For example, 26 from the 49 VRs in mathematics were identified to possess abstract mathematical meaning, while the relative frequency of this category in mathematics texts is almost 53%. Finally, only 4% of all VRs in mathematics were identified to express physical meaning.

Table 4.6: The frequencies of categories of VRs in relation to contextual meaning

Context		Mathematics N=49 %	Astronomy N=23 %	Physics N=101 %	Applied technologies N=41 %
Physical Meaning	Every day life (C1)	4,08	0,00	24,75	0,00
	Natural life (C2)	0,00	95,65	3,96	0,00
	Scientific devices (C3)	0,00	0,00	33,66	39,02
Mathematical Meaning	Abstract (CM1)	53,06	4,35	5,94	4,88
	Semi-abstract (CM2)	8,16	21,74	44,55	70,73
	Applied (CM3)	38,78	43,48	0,00	4,88

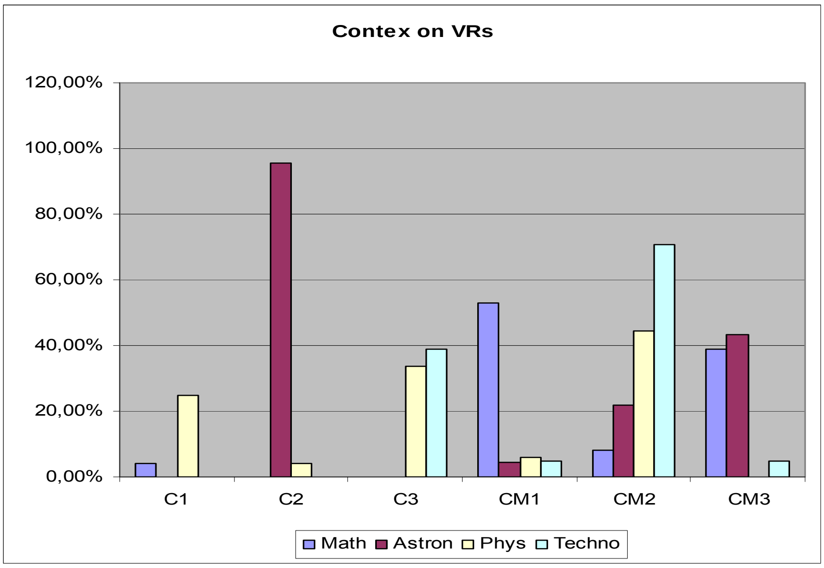


Figure 4.12: The figural representations of Table 4.4 data in the different subjects

Issues emerging from Table 4.3 and Figure 4.12

VRs identified as possessing physical meaning based on every day examples are met mostly in the subject of Physics. Almost one out of four VRs in Physics are falling in this category.

VRs whose context is identified as having physical meaning based on Natural life is met mostly, as it was expected, in Astronomy. Almost all VRs in Astronomy fall in this category.

VRs whose context is identified as physical meaning based on scientific devices is only met in the subject of Physics and Applied Technologies.

95% of all visual representations in mathematics texts are free of any physical meaning. So, mathematics texts on their own seem to fail in helping students make connections between periodic phenomena of everyday life and/or the natural world and abstract mathematical notions.

On the other hand, one out of two VRs (53%) in mathematics texts direct the reader towards abstract mathematical meaning. The other subjects participate in this subcategory with very little proportions.

VRs whose context is identified as possessing semi-abstract mathematical meaning could be of all categories but mostly in applied technologies (70%) and physics (45%). The VRs in these subjects are usually images of time-dependent periodic processes.

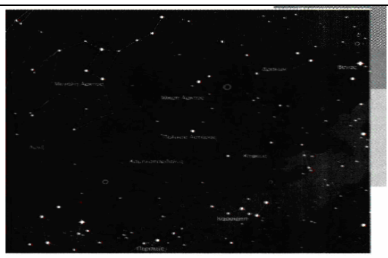
Finally, VRs whose context is identified as possessing applied mathematical meaning (usually Table value charts) are very common in Mathematics and Astronomy.

Finally, since not all VRs possess at the same time mathematical and physical meaning, it seems that the category of VRs carrying mathematical meaning is more favourable among authors in school texts. This means that students must mostly use mathematical tools to interpret these images.

4.3c) The category of VRs Genre

4.3c1): The subcategory of Photographs

Photographs depict the physical and natural appearance of reality. In Table 4.7 we present the analysis of examples of Photographs (every day life instances; Natural life; Chrono-photography; and scientific device) from the subject of Physics and Astronomy. The other subjects do not participate in this category.

Table 4.7. Examples of Photographs	
VRs	School Subject & Grade level Analysis
 **Εικ. 1.3** Στη φωτογραφία φαίνονται παιδιά να κάνουν κούνια. Όταν η απομάκρυνση είναι μέγιστη, η ταχύτητα είναι μηδενική.	Physics Grade 12b *Context* Physical meaning: Every day life Mathematical meaning: - *Genre* Photo: Every day life instances
 ***Εικόνα 2.10:*** Οι αποστάσεις μεταξύ των αστέρων φαίνονται αμετάβλητες από έναν επίγειο παρατηρητή. Πρόκειται για μια ακόμα ψευδαίσθηση. Οι σχετικές τους ταχύτητες είναι πολύ μεγάλες. Επειδή, όμως οι αποστάσεις τους από τη Γη είναι τεράστιες, η γωνιακή τους μετατόπιση μέσα σε μια ανθρώπινη ζωή είναι αδύνατο να γίνει αισθητή.	Astronomy, Grade 11 *Context* Physical meaning: Natural life Mathematical meaning: - *Genre* Photo: Natural life instances
Εικ. 1.2 Διαδοχικά στιγμιότυπα της ταλάντωσης σφαίρας εξαρτημένης από ελατήριο. Το χρονικό διάστημα ανάμεσα σε δύο διαδοχικά στιγμιότυπα είναι σταθερό. Στη διάρκεια της φωτογράφησης η φωτογραφική πλάκα μετατοπίζεται οριζόντια με σταθερή ταχύτητα. Έτσι η φωτογραφία δείχνει πως μεταβάλλεται η κατακόρυφη απομάκρυνση σε συνάρτηση με το χρόνο. 	Physics Grade 12b *Context* Physical meaning: Every day life Mathematical meaning: Applied *Genre* Photo: Chrono-photography
 **Εικ. 3.4** Παλμογράφος. Photo 3.4 Oscilloscope	Physics Grade 11b *Context* Physical meaning: Scientific devices Mathematical meaning: Applied *Genre* Photo: Technological devices

Counting of relative frequencies of appearance of the final produced schemes in the form of systemic network of the dimension of Genre and the subcategory of photographs on the VRs analyzed in each subject gave us Table 4.8 and the histogram Figure 4.13.

Table 4.8: The relative frequencies of VRs in relation to their Genre: The subcategory of photographs

Photos	Mathematics N=49 %	Astronomy N=23 %	Physics N=101 %	Applied technologies N=41 %
Every day life instances (Ph1)	0,00	0,00	10,89	0,00
Natural life instances (Ph2)	0,00	8,70	0,99	0,00
Chrono-photography (Ph3)	0,00	0,00	3,96	0,00
Technological devices (Ph4)	0,00	0,00	6,93	0,00

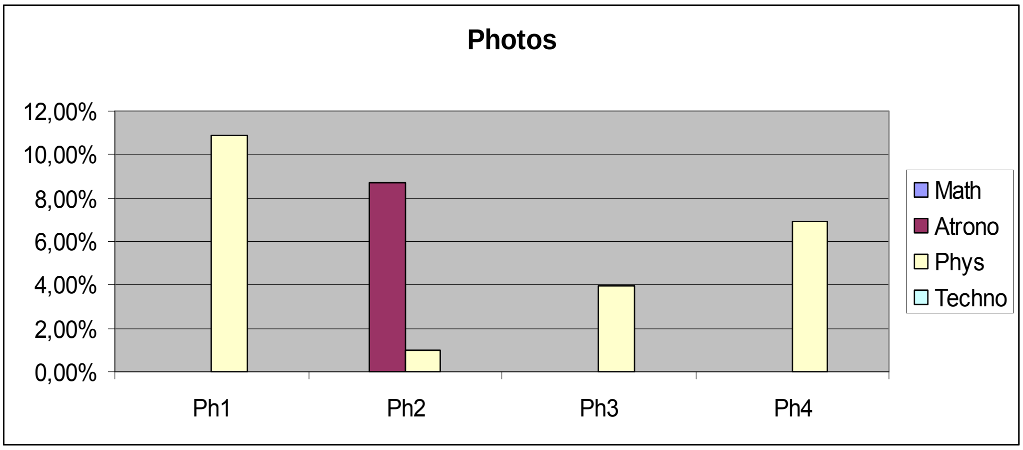


Figure 4.13: The Figural representations of Table 4.8 data in the different subjects

Issues emerging from Table 4.8 and Figure 4.13.

A general observation is that the subject of mathematics and applied technology are completely absent in this category. On the contrary, in the subject of physics photos are very popular VRs since 20% of all VRs in physics texts are photographs, mostly images from every day life but scientific devices and chrono-photographies as well.

In Astronomy texts, almost one out of ten VRs are images of natural life.

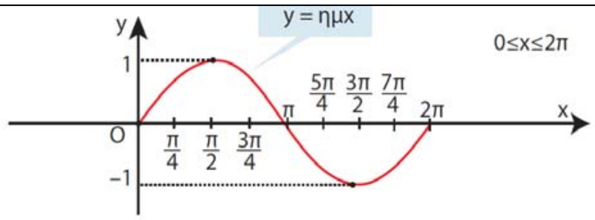
Why do textbook authors in the subject of Mathematics avoid using photographs? Maybe because they think that a photography depicts reality and lacks any mathematical meaning. But chrono-photography in Table 4.7 has a physical and a mathematical meaning.

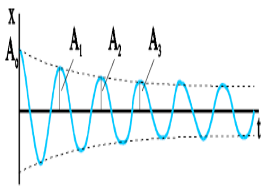
Finally, it is an open question why authors in applied technology texts avoid using photographs.

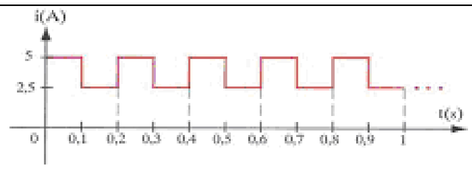
4.3c2) The subcategory of Graphs

Many graphical representations are met as images in all texts analyzed. In Table 4.9 we present some examples of graphical representations and their analysis from the subjects of Mathematics, Physics and Applied Technology.

<table>
<tr><td colspan="2">Table 4.11 Examples of graphs analysis</td></tr>
<tr><td>VRs</td><td>School Subject & Grade level

Analysis</td></tr>
<tr><td></td><td>Mathematics Grade 11

Context
Physical meaning: -
Mathematical meaning: Abstract
Genre
Graph: Periodic motion; sinusoidal function</td></tr>
<tr><td>
Σχ. 1.21 Σε μια φθίνουσα ταλάντωση ο λόγος των διαδοχικών μέγιστων είναι σταθερός.</td><td>Physics Grade 12b

Context
Physical meaning: -
Mathematical meaning: semi-abstract
Genre
Graph: Repeated but not periodic function</td></tr>
<tr><td></td><td>Applied technology, Grade 11, Electrology

Context
Physical meaning: -
Mathematical meaning: semi-abstract
Genre
Graph: Periodic functions; other.</td></tr>
</table>

Counting of relative frequencies of appearance of the final produced schemes in the form of systemic network for the dimension of graphical representations on all the Visual images in each subject gave us Table 4.10 and the histogram representation in Figure 4.14.

<table>
<tr><td colspan="6">Table 4.10. The relative frequencies of VRs in relation to their Genre: The subcategory of graphs</td></tr>
<tr><td colspan="2">Graphs</td><td>Mathematics
N=49
%</td><td>Astronomy
N=23
%</td><td>Physics
N=101
%</td><td>Applied technologies
N=41
%</td></tr>
<tr><td rowspan="2">Periodic functions</td><td>Sinusoidal (Sin)</td><td>20,41</td><td>0,00</td><td>15,84</td><td>26,83</td></tr>
<tr><td>Sum of sinusoidal (sum_sin)</td><td>2,04%</td><td>0,00</td><td>1,98</td><td>2,44</td></tr>
</table>

	Other periodic (oth_per)	8,16%	0,00	1,98	14,63
Repeated but non-periodic (rep_non_per)		0,00%	0,00	2,97	2,44
Other mathematical relations (non_per)		2,04%	4,35	5,94	9,76

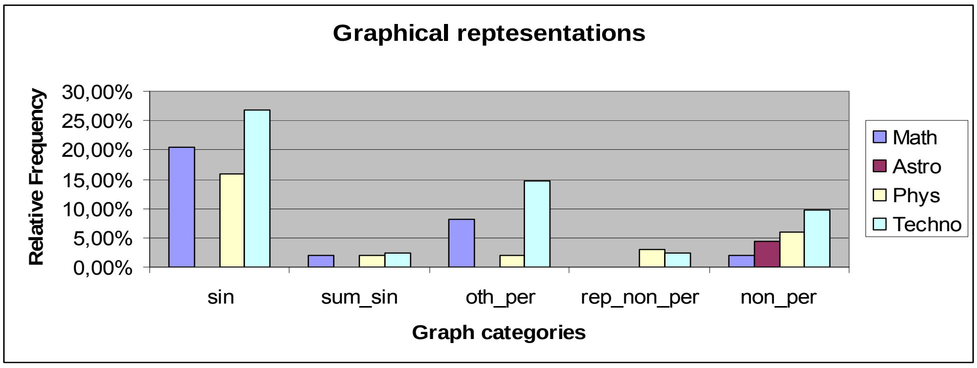


Figure 4.14: The figural representations of Table 4.8 data in the different subjects

Issues emerging from the Table 4.10 and Figure 4.14

The main category that is the most popular in all texts of all subjects, except for Astronomy, is that of sinusoidal functions. It seems that sinusoidal are more favourable visual images in applied technology texts (27%) than in Mathematics texts (20%). We have to mention that although the graphs of sine and cosine functions are sinusoids in different phases, authors prefer to use graphs of the sine rather than the cosine function.

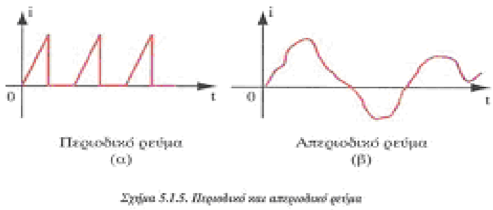
The subcategory of other periodic graphs mostly appears in Applied Technology texts and in Mathematics texts. In mathematics texts, graphs are usually about trigonometric functions (other than sine and cosine), while in Applied Technology texts graphs of periodic currents are not presented in a sinusoidal form.

The subcategory of graphs that represent other mathematical relations (e.g. linear functions) appears in all subjects but in a very low rate.

Graphs that represent a sum of sinusoidal functions are extremely rear in texts (2%-2.5%) This might be a problem for students since these types of graphs could motivate students to acquire broad knowledge concerning the periodical attitude of graphs since these graphs are not sinusoidal in general.

Moreover, the category of repeated but not periodic function sparsely appears in all texts from all subjects while it is completely absent in mathematics. The author could use this type of representations as non- examples of periodic functions. Only one such case was identified in an applied technology text (Electrology) and is presented in Figure 4.15 This type of representations could serve as to clarify boundaries between periodical and non-periodical behaviours.

Finally, graphical images of damped oscillations in physics are not specifically presented as repeated but not periodic graphical representations and this could contribute to confusions.

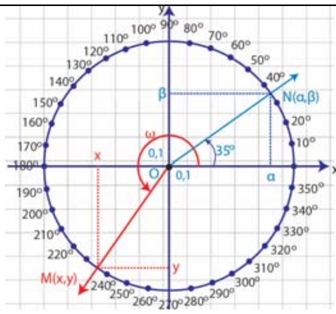
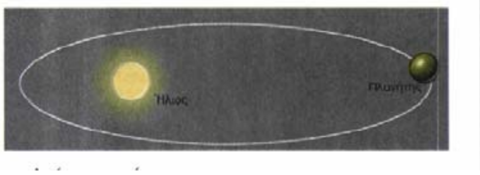


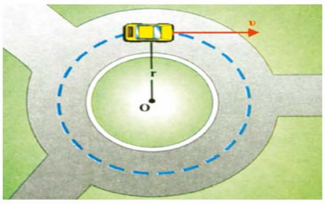
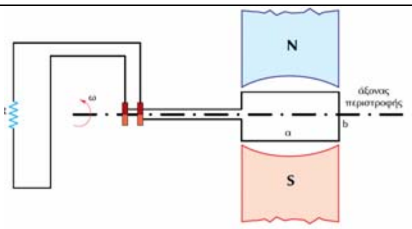
(a) Periodic current (b) non-periodic current.

Figure 4.15: An example and a non-example of a periodic function.

4.3c3) The subcategory of schematic representations

The last category of VRs genre is the schematic representations. In Table 4.11 we present some examples of schematic representations and their analysis from the subjects of Mathematics, Astronomy, Physics and Applied Technology.

Table 4.11: Examples of schematic representations and their analysis	
VRs	School Subject & Grade level Analysis
	Mathematics Grade 11 *Context* Physical meaning: - Mathematical meaning: Applied *Genre* Schematic representation/ Periodic models/ circular motions/ Trigonometric circle
	Astronomy, Grade 11 *Context* Physical meaning: Natural life Mathematical meaning: Applied *Genre* Schematic representation/ Periodic models/ elliptical motion/ Specific

 *Το αυτοκίνητο κινείται στην κυ-κλική πλατεία με σταθερή ταχύ-τητα.* **Εικόνα 1-4.**	Physics, Grade 11a *Context* Physical meaning: Every day life Mathematical meaning: Applied *Genre* Schematic representation/Periodic models/ circular motion/Specific
 Σχήμα 9.3. *Περιστροφή πλαισίου για παραγωγή εναλλασσόμενου ρεύματος*	Applied Technology, Grade 11 (Electronics) *Context* Physical meaning: Scientific devices Mathematical meaning: Semi- abstarct *Genre* Schematic representation/Periodic models/ Electrical/Rotating frame

Counting relative frequencies of appearance of the final produced schemes in the form of systemic network for the dimension of schematic representations on the VRs analyzed in each subject gave us Table 4.10 and the histogram Figure 4.8

Table 4.12. The relative frequencies of VRs in relation to their Genre: The subcategory of schematic representations

Schematic representations			Mathematics N=49 %	Astronomy N=23 %	Physics N=101 %	Applied technologies N=41 %
Periodic models	Circular motions	Trigonometric circle (Ci1)	18,37	0,00	0,00	2,44
		Vectors/complex numbers (Ci2)	8,16	0,00	0,00	34,15
		Specific (Ci3)	4,08	39,13	8,91	0,00
		Abstract (Ci4)	2,04	0,00	0,00	0,00
	Elliptical motion	Specific (El1)	0,00	13,04	0,00	0,00
		Abstract (El2)	0,00	0,00	0,00	0,00
	Mechanical oscillation	Spring(Sr)	0,00	0,00	10,89	0,00
		Pendulum (P)	0,00	0,00	7,92	0,00
	Electrical	Rotating frame (Rot)	0,00	0,00	1,98	29,27
		Circuit	0,00	0,00	3,96	7,32
	Constructed situations (Con)		4,08	21,74	9,90	0,00
Tables			20,41	13,04	4,95	0,00

Celestial bodies (Cel)	0,00	39,13	0,00	0,00
Other representations (Other)	22,45		12,87	9,76

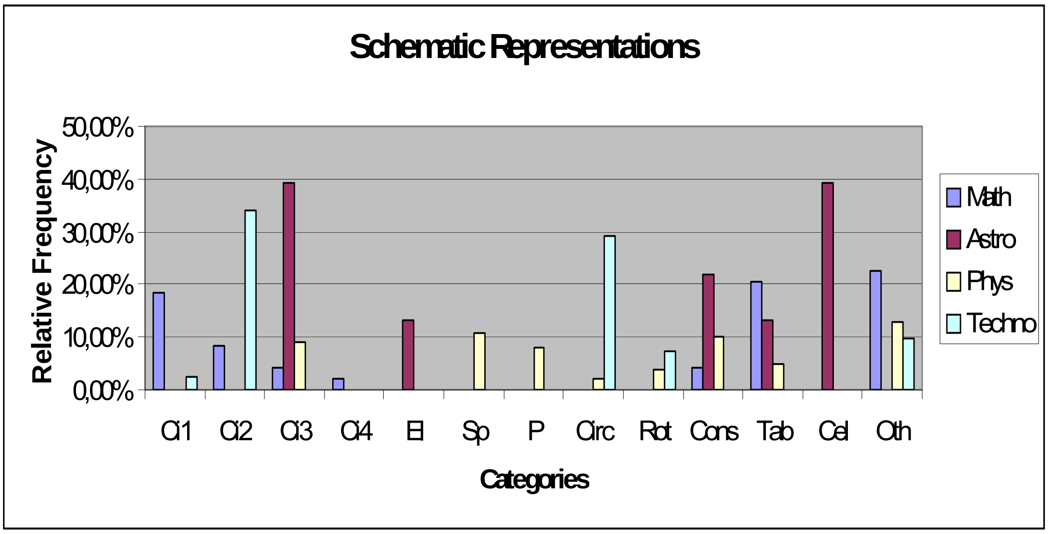


Figure 4.16: The figural representations of Table 4.12 data in the different subjects.

Issues emerging from the observation of Table 4.12 and Figure 4.16

By observing the histogram representation, someone is led to the conclusion that every subject uses different types of schematic representations according to their goals and activities.

In the subject of mathematics the most popular schematic representations are other representations e.g. triangles or the Cartesian plane the tables and the Trigonometric circle. Divergence in schematic representations of periodic motions in the different subjects could contribute to a rich foundation of concept images.

The most popular schematic representations in the subject of astronomy are specific circular motions and figures of celestial bodies.

In the subject of physics the most popular schematic representations are other representations (e.g. triangles); the spring and the constructed situations. Constructed situations are images of periodic motions that could contribute to students' meanings.

In the subject of Applied Technology the most popular schematic representations are circuits (as expected) and vectors rotating circular motions which came as a surprise.

The schematic representations of spring and pendulum are very frequent in physics, but are absent in all other courses.

The schematic representation of a rotating frame appears in some cases in applied technologies and physics. Constructed periodic schematic representations are common in Astronomy, and Physics.

4.4) The co-deployment of VRs and modes of reasoning

The content analysis revealed the interplay between the visual and the verbal components of a school text in the thematic unit. Since this may play a role on students' conceptualization, we analyzed the co-deployment of VR and verbal text.

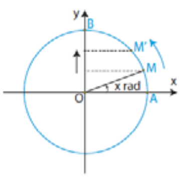
In the section 4.4a we present the scheme developed and we exemplify our analysis in two examples from the subjects of Mathematics and Physics. We spot issues that emerge from our cross subject analysis, while in section 4.4b we present the results of our quantitative analysis and the issues emerging.

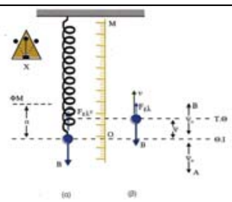
4.4a) Qualitative analysis: The case of mathematics and physics text

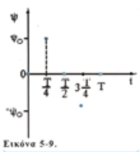
After analyzing many texts, we developed a the scheme for the functions of VR in the text's reasoning. Seven mutually exclusive categories have been identified: (a) *illustrative* (the VR adds to the verbal component without being embedded in the reasoning); (b) *exemplifying*; (c) the VR is the *starting point* on which reasoning is developed; (d) the VR is the *fundamental* tool in the reasoning of the etxt; (e) The VR is the *product* of the reasoning; (f) The VR *organizes* the outcome of the reasoning; and (g) *Complementary* (the content of the VR complements the reasoning of the verbal text).

We exemplify our analysis in two texts. In table 4.13, we present the two texts (in Greek), while in Table 4.13a we present the visual components separately and in Table 4.13b we present the verbal (in English) components that support the analysis of the visual components.

<table>
<tr><td colspan="2">Table 4.13</td></tr>
<tr><td>Mathematics text</td><td>Physics text</td></tr>
<tr>
<td>
Μελέτη της συνάρτησης f(x) = ημx

Επειδή η συνάρτηση f(x) = ημx είναι περιοδική με περίοδο 2π, αρκεί να τη μελετήσουμε σε ένα διάστημα πλάτους 2π, π.χ το [0,2π]. Έχουμε αναφέρει όμως ότι το ημx είναι η τεταγμένη του σημείου M στο οποίο η τελική πλευρά της γωνίας xrad τέμνει τον τριγωνομετρικό κύκλο. Επομένως αρκεί να εξετάσουμε πώς μεταβάλλεται η τεταγμένη του M, όταν αυτό περιφέρεται στον τριγωνομετρικό κύκλο κατά τη θετική φορά, ξεκινώντας από το A.
Παρατηρούμε ότι:
• Όταν το x μεταβάλλεται από το 0 μέχρι το $\frac{\pi}{2}$, το M κινείται από το A μέχρι το B. Άρα η τεταγμένη του αυξάνει, που σημαίνει ότι η συνάρτηση f(x) = ημx είναι γνησίως αύξουσα στο διάστημα $\left[0,\frac{\pi}{2}\right]$. Ομοίως βρίσκουμε ότι η συνάρτηση f(x) = ημx είναι:


76 ΚΕΦΑΛΑΙΟ 3ο: ΤΡΙΓΩΝΟΜΕΤΡΙΑ
— γνησίως φθίνουσα στο διάστημα $\left[\frac{\pi}{2},\pi\right]$
— γνησίως φθίνουσα στο διάστημα $\left[\pi,\frac{3\pi}{2}\right]$ και
— γνησίως αύξουσα στο διάστημα $\left[\frac{3\pi}{2},2\pi\right]$
</td>
<td>


Με τη βοηθεια του χρονομετρου βρισκουμε την περιοδο Τ της ταλάντωσης μετρώντας το χρόνο για τη διαδρομή ΑΟΒΟΑ ή για τη διαδρομή ΟΒΟΑΟ ή για οποιονδήποτε «κύκλο» και διαπιστώνουμε ότι παραμένει σταθερή.
Μπορούμε επίσης να μετρήσουμε τους χρόνους για τις διαδρομές ΑΟ, ΟΒ, ΒΟ και ΟΑ και να διαπιστώσουμε ότι είναι ίσοι μεταξύ τους (άρα ο καθένας είναι ίσος με Τ/4).
<table>
<tr><th>ψ</th><th>t</th></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td>ψ_0</td><td>T/4</td></tr>
<tr><td>0</td><td>T/2</td></tr>
<tr><td>$-\psi_0$</td><td>3T/4</td></tr>
<tr><td>0</td><td>T</td></tr>
</table>
Εικόνα 5-8.
Πίνακας τιμών της απομάκρυνσης σε χαρακτηριστικές χρονικές στιγμές.
Με τη βοήθεια των μετρήσεων που μέχρι τώρα έχουμε κάνει μπορούμε να συμπληρώσουμε ένα πίνακα τιμών (Εικ. 8) της απομάκρυνσης ψ σε συνάρτηση με το χρόνο κίνησης t (για απλούστευση θεωρούμε μηδέν τη χρονική στιγμή που το σώμα περνά από τη Θ.Ι.) και να σχεδιάσουμε με τη βοήθειά του την καμπύλη ψ = f(t) (Εικ. 9). Όμως τόσο ο πίνακας όσο και το διάγραμμα, μας δίνουν πολύ λίγες πληροφορίες.
Αν θέλουμε οι πληροφορίες αυτές να είναι πολύ περισσότερες, μπορούμε, αν φυσικά έχουμε τη δυνατότητα, να χρησιμοποιήσουμε χρονοφωτογραφία όπου το σώμα στη διάρκεια μίας περιόδου έχει φωτογραφηθεί πολλές φορές σε διάφορες θέσεις.


Εικόνα 5-9.
</td>
</tr>
</table>

• Η συνάρτηση παρουσιάζει

— μέγιστο για $x=\frac{\pi}{2}$, το $ημ\frac{\pi}{2}=1$ και

— ελάχιστο για $x=\frac{3\pi}{2}$, το $ημ\frac{3\pi}{2}=-1$.

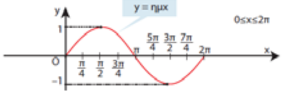
Τα συμπεράσματα αυτά συνοψίζονται ως εξής:

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
ημx	0 ↗	1 μεγ. ↘	0 ↘	−1 ελαχ. ↗	0

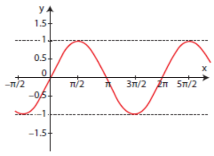
Για να κάνουμε τη γραφική παράσταση της συνάρτησης χρειαζόμαστε έναν πίνακα τιμών της. Κατά τα γνωστά έχουμε:

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
ημx	0	$\frac{\sqrt{2}}{2}=0,71$	1	0,71	0	−0,71	−1	−0,71	0

Παριστάνουμε με σημεία του επιπέδου τα ζεύγη αυτά των αντίστοιχων τιμών και τα ενώνουμε με μια συνεχή γραμμή.
Έτσι προκύπτει η παρακάτω γραφική παράσταση της συνάρτησης ημίτονο στο διάστημα [0, 2π]:



Επειδή η συνάρτηση f(x) = ημx είναι περιοδική, με περίοδο 2π, η γραφική της παράσταση έχει την ίδια μορφή στα διαστήματα [2π, 4π], [4π, 6π] κτλ. καθώς και στα διαστήματα $[-2\pi,0],[-4\pi,-2\pi]$ κτλ.
Έτσι έχουμε την ακόλουθη γραφική παράσταση της συνάρτησης ημίτονο, η οποία λέγεται **ημιτονοειδής καμπύλη**.



Τέλος γνωρίζουμε ότι οι αντίθετες γωνίες έχουν αντίθετα ημίτονα. Άρα για κάθε $x \in \mathbb{R}$ ισχύει $ημ(-x)=-ημx$. Αυτό σημαίνει ότι η συνάρτηση f(x) = ημx είναι περιττή και επομένως η γραφική της παράσταση έχει κέντρο συμμετρίας την αρχή 0(0,0) των αξόνων.

Αυτές οι θέσεις απέχουν χρονικά μεταξύ τους όσο ο χρόνος μεταξύ δύο διαδοχικών φωτογραφίσεων (η απομάκρυνση μετριέται με την μετροταινία που επίσης φαίνεται στις φωτογραφίες).

Έτσι ο πίνακας τιμών ψ-t είναι αρκετά πλήρης ώστε η καμπύλη ψ=f(t) που με τη βοήθειά του κατασκευάζουμε (Εικ. 10) να μπορεί να σχεδιασθεί συνεχής και να θεωρείται ότι βρίσκεται πολύ κοντά στην πραγματική.

Αυτή η καμπύλη έχει ημιτονοειδή μορφή πράγμα που είναι και το χαρακτηριστικό της γραμμικής αρμονικής ταλάντωσης.

Γραμμική αρμονική ταλάντωση λέγεται η ταλάντωση που πραγματοποιεί ένα σώμα όταν η τροχιά του είναι ευθεία γραμμή και η απομάκρυνση του ημιτονοειδής συνάρτηση του χρόνου.

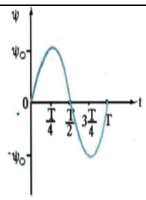
(η ημιτονοειδής συνάρτηση λέγεται και αρμονική).

Την καμπύλη ψ = f(t) που προηγουμένως κατασκευάσαμε μπορούμε να δούμε άμεσα αν τροποποιήσουμε το πείραμα που εκτελέσαμε προσαρτώντας μία γραφίδα στο σώμα, η άκρη της οποίας μόλις ακουμπά στο χαρτί μιλλιμετρέ με το οποίο είναι καλυμένη η παράπλευρη επιφάνεια κυλίνδρου που περιστρέφεται με σταθερό ρυθμό γύρω από τον άξονά του (Εικ. 11).

Μπορούμε μάλιστα να μετρήσουμε με τη βοήθεια της καμπύλης την απομάκρυνση για διάφορες χρονικές στιγμές και να κατασκευάσουμε τον πίνακα τιμών ψ-t.

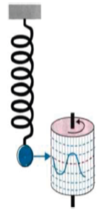
(Είναι προφανές ότι για να μην αποτυγχάνει αυτό το τροποιημένο πείραμα πρέπει η περίοδος περιστροφής του κυλίνδρου να είναι μεγαλύτερη από την περίοδο του σώματος και το πλάτος της ταλάντωσης μικρότερο από το μισό του ύψους του κυλίνδρου).

β. Εξισώσεις κίνησης



Η απομάκρυνση είναι ημιτονοειδής συνάρτηση του χρόνου.
Εικόνα 5-10.

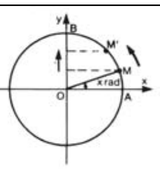
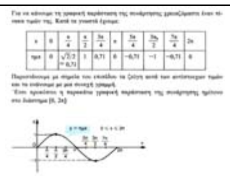
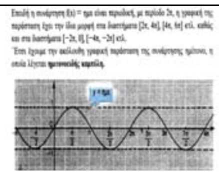
Το σώμα που πραγματοποιεί Γ.Α.Τ. λέγεται και αρμονικός ταλαντωτής.

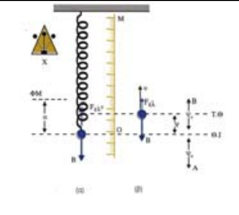
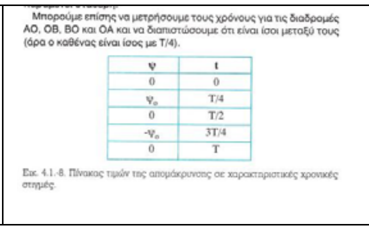
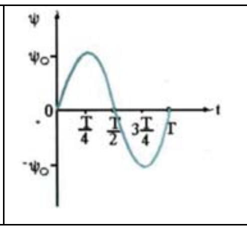
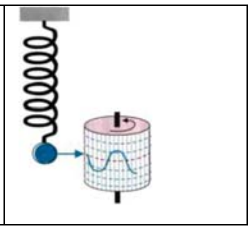


The mathematical text is taken from the subject of trigonometry and its thematic content is: "Graphing the sinx function".

The science text is from the subject of Oscillations and its thematic content is: "Defining the Linear Harmonic oscillation".

Both texts study the function that models a specific periodic phenomenon. Particularly, in the mathematical text the periodic phenomenon is the rotation of a point M that moves counterclockwise on the unit circle (VR1m), while in the science text a body is presented that oscillates with the help of a spring (VR1sc). The function that models both phenomena is the sinusoidal function.

Table 4.13a: THE VISUAL COMPONENTS OF THE MATHEMATICAL TEXT			
VR1m	VR2m	VR3m &VR4m	VR5m
			
THE VISUAL COMPONENTS OF THE SCIENCE TEXT			
VR1sc	VR2sc	VR3sc	VR4sc

In the Table 4.13a, we can identify two models of periodic motions (the rotational in the mathematical text (VR1m) and the oscillatory in the science text (VR1sc); three value charts (VR2m, VR3m & VR2 sc); three sinusoidal curves (VR4m, VR5m & VR3sc); and one scheme which integrate the above two periodic motions and the sinusoidal curve (VR4sc).

By analysing the verbal mode we could identify four common actions in the development of the presented conceptual field throughout each text. Action 1 refers to how the period T is defined, while Action 2 refers on how each periodic phenomenon is studied. Action 3 refers to the generalized outcomes of Action 2 and Action 4 refers to how the authors apply the new knowledge on specific situations. Action 1, 3 and 4 are comprehended through one mode of reasoning (not always the same in each text), while Action 2 is accomplished through two modes of reasoning in the mathematical and one in the science text. In the following table (Table 4.13b) we present the four actions in the argumentation process, we comment on the modes of reasoning developed and on the visual-verbal relations.

<table>
<tr><td colspan="2">Table 4.13b: The Verbal component of the two texts and our analysis</td></tr>
<tr><td>MATHEMATICS TEXT</td><td>SCIENCE TEXT</td></tr>
<tr><td colspan="2">Action 1: Define the interval T to study the periodic motion</td></tr>
<tr><td>"Since the function f(x)=sinx is periodic with period 2π it is sufficient to study it in an interval that has length 2π, e.g. [0, 2π]".

We consider this mode of reasoning as Nomo-logical (N1) since a previously known property of the sine function consists the basis for defining the period to study the phenomenon.</td><td>"In order to study the oscillation that an object conducts with the help of a spring we need an ideal spring, a compact object, a timer and a tape measure. With the help of a timer we find the T period of the oscillation counting the time for every 'cycle' of route (e.g. AOBOA or OBOAO) and note that it remains constant".

We consider this mode of reasoning as Empirical (E2) since it describes an enactive experience of an experimental activity. The justification proposed is made with the help of empirical measurements.

The visual representation (VR1sc) is the starting point on which this mode of reasoning is developed.</td></tr>
<tr><td colspan="2">Action 2: Study the periodic motion on the interval of period T.</td></tr>
<tr><td>By reminding the reader that sinx represents the y-coordinate of the point M(x,y) on the unit circle continues: "We notice that as x values from 0 to π/2 the</td><td>On the basis of experimental measurements defines the time interval T/4 and invites the reader to record the displacement y(t) in each point. The</td></tr>
</table>

<table>
<tr>
<td>point M moves from A to B. Therefore, the y-coordinate increases, thus the function sinx is strictly increasing in the interval [0, $\pi/2$]. Similarly, we find that the function is strictly decreasing in the interval [$\pi/2$, π]. [...] Moreover, the function has a maximum value on $x=\pi/2$ (sinx=1) and a minimum value on $x=3\pi/2$ (sinx=-1). The results are summarized in the following table" (VR2m).

Reasoning in this case is based on a model (the unit circle) and since it starts from actual data (monitoring the y-coordinate of the point M) and ends in general phrasing as presented in VR2m we classify it as logical-empirical (LE2).

The role of VR1m in this mode of reasoning is fundamental since the reader must reflect on this VR throughout the thinking process. The table representation (VR2m) organizes the 'steps' of the reasoning.</td>
<td colspan="2" rowspan="2">recorded data are presented in an abstract form on VR2sc. "If we want this information to be more precise we can use chronophotography where the body motion has been photographed several times in different positions during one period. Thus, the value chart is quite thorough so as for the y=f(t) curve (VR3sc) to be designed continuously and assume that it is very close to the real one. This curve has a sinusoidal form which is the characteristic feature of the linear harmonic function".

Although the reasoning is based on experimental methods, there is an obvious intention to generalize these outcomes in the last sentence. Moreover, we can trace generalized semiotic elements in the VRs (in the value chart and in the graph representations e.g. ψ_o). These elements led us to discern this reasoning from the empirical ones and characterize it as Logical- empirical (LE2).

VR2sc presents how the reader could organise the experimental outcomes while the curve VR3sc is the product in this mode of reasoning.</td>
</tr>
<tr>
<td>The text continues by employing mathematical relations (presented on VR3m) in order to sketch the graph of sinx in the interval [0, 2π] (i.e. VR4m).

We characterize this mode of reasoning as logical - mathematical (LM) while the tables VR2m and VR3m are the starting point in this reasoning and the sinusoidal curve (VR4m) is the product of this mode of reasoning.</td>
</tr>
<tr>
<td colspan="3">Action 3: Generalization (Nomo-logical (N2) mode of reasoning since the definition emerges as a result of previous inferences)</td>
</tr>
<tr>
<td>"Since the function f(x)=sinx is periodic, with period 2π, the curve has the same shape in the intervals [-2π, 0] [...]. So, we have the following graph which is called sinusoidal function (VR5m)".

Now VR4m is the initial situation, the starting point, while VR5m is the product.</td>
<td colspan="2">"Linear Harmonic oscillation is the oscillation that an object performs when its orbit is on a straight line and its displacement is a sinusoidal function of time"

In this case VR4sc has an illustrative character.</td>
</tr>
<tr>
<td colspan="3">Action 4: Applying the new knowledge in specific situations (LE1 mode of reasoning)</td>
</tr>
</table>

"We know the opposite angles have opposite sines. Hence, for every x ∈ R sin(-x) =-sinx. This means that the function is odd and hence the graph has a point symmetry on 0(0,0)". In this mode of reasoning VR5m has an *explanatory* role.	Supports the definition by providing another example of a modified experiment as follows: *"If we modify the experiment, we can directly see the curve that we have previously formed if we adjust a stylus to the object* [...] (VR4sc)". VR4sc *exemplifies* the mode of reasoning developed.

Issues emerging

The case of modes of reasoning: Our analysis on the one side identifies common modes of reasoning (when empirical data are the basis for logical conclusions or when exemplifying these conclusions on specific situations). Furthermore, these common modes of reasoning are impended in the goals of each subject they could establish common reasoning behaviours in the different educational fields. On the other side, we identified modes of reasoning in the mathematical text (Nomo-logical (N1) and Logical-mathematical (LM)) that are absent in the science text. This absence influences the argumentation process since additional examples are provided to persuade and convince the reader on the produced knowledge. These differences in argumentation are not only identified in the specific examples but throughout our extended data. Besides, the presence of different routes in reasoning when defining the period T (nomo-logically (N1) i.e. relying on previously established statements and experimentally (E2) i.e. making genuine links to the real world) could motivate and allow students to broaden and enrich their perception of this notion.

The case of visual-verbal relation: By analyzing the visual-verbal relation on each mode of reasoning we recognize that common types of Visual Representations serve common purposes in different modes of reasoning (e.g. the sinusoidal curves as reasoning products or the tables as organizing tools). Furthermore, our analysis illustrates how the visual representations change their character in subsequent modes of reasoning (e.g. from product to the starting point of reasoning). This changing character on the one hand contributes to the cohesion of the text argumentation while on the other hand highlights the 'flexible' character of the visual components. Moreover, the visual product of argumentation in mathematics (the sinusoidal curve) acts as a prototypical image of the sine function in the science text. This piece of evidence establishes the freedom of the mathematical visual images to travel across subjects and possibly contribute to the formation of the invariant notion of periodicity as described by Vergnaud (2009).

4.4b) Quantitative analysis

Counting of relative frequencies of VRs appearance in terms of the produced scheme across subjects gave us the numbers presented in Table 4.14 and in the histogram Figure 4.17

Table 4.14 The relative frequencies of the co-deployment of VRs and reasoning

Categories on visual-verbal relation	Mathematics N=49 %	Astronomy N=23 %	Physics N=101 %	Applied technologies N=41 %
Illustrative (Il)	0,061	0,000	0,000	0,000
Explanatory (Ex)	0,041	0,130	0,267	0,171
Starting point(Str)	0,367	0,304	0,248	0,366
Fundamental (FUN)	0,429	0,043	0,158	0,195
Product (PR)	0,204	0,217	0,208	0,268
Organizing tool (ORG)	0,082	0,000	0,099	0,049
Complementary (Com)	0,000	0,000	0,010	0,000

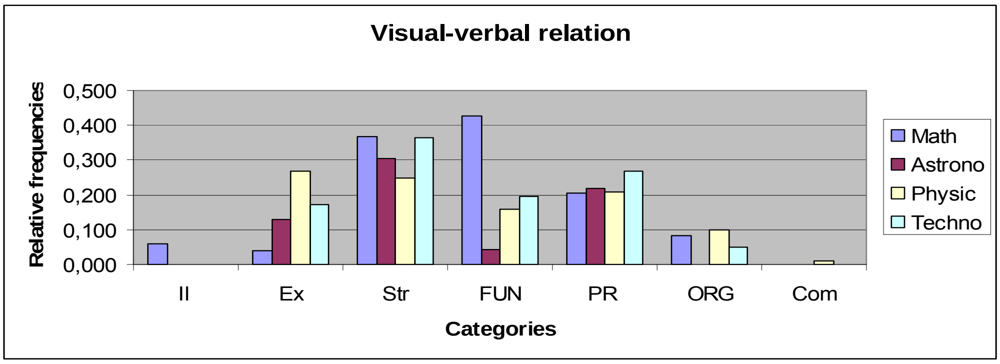


Figure 4.17: The figural representations of Table 4.14 data in the different subjects.

Issues emerged

The study of the co-deployment of visual representations and reasoning in texts shows divergent behaviors in the different subjects.

In mathematics texts VRs seem to undertake a significant role acting as fundamental tools (42,9%), starting points (36,7%), or as the product of reasoning (20%). On the contrary, in mathematics texts images as examples or illustrations are rarely used.

Astronomy texts use images in their reasoning as the starting point (30%), or the product (22%) of their rationale, or as examples (13%).

Physics texts use VRs mostly as the basis of explanatory reasoning (27%) or as the starting point (25%) of reasoning or as the product of reasoning (20%) or as a fundamental tool (16%).

Applied technology texts use images mostly as the starting point (38%) or as the product of reasoning (27%) and as the fundamental tool in reasoning (20%).

Finally, almost 8% of the VRs in physics play a complementary role. Their content adds further information to that of the verbal text.This category is absent in mathematics.

The above differences in the co-deployment of visual representation and reasoning are not necessarily conflicting, but complimentary, for a scientist or a mathematician. But students need to be supported in order to make the connections and fill in the existing gaps between different visual-verbal relations among subjects. This fact stresses responsibility on the part of educators in being efficient to handle this task.

4.5) Implementation on the dimension of the proposed exercises

The problem solving activities found in textbooks are an important tool for students' practice with the conceptual field and they also participate in students' conceptualization (students understand their capabilities and limits of their own thought processes; acquire heuristics, awareness and management etc.). So we decided to analyzalso the proposed exercises (as our main units of analysis) and the individual questions as well.

Particularly, we analyzed the exercises proposed by the authors in all subjects (apart from Astronomy) on the selected textbooks chapters. We excluded the cases in which we restricted the analysis on selected pages of a textbook (e.g. Physics Grade 11b) and not on a whole chapter.

4.5a) Qualitative analysis

The systemic network produced after the analysis of 171 exercises in mathematics, physics and applied technologies texts is presented in Figure 4.18.

The BAR ([) notation signifies that all the categories are mutually exclusive, whereas the BRA ({) notation signifies that any number or even all of the categories can be selected simultaneously.

It was found that the proposed exercises can be analyzed according to three dimensions: the *demands of exercises*, the role of VRs, and their type of *context*.

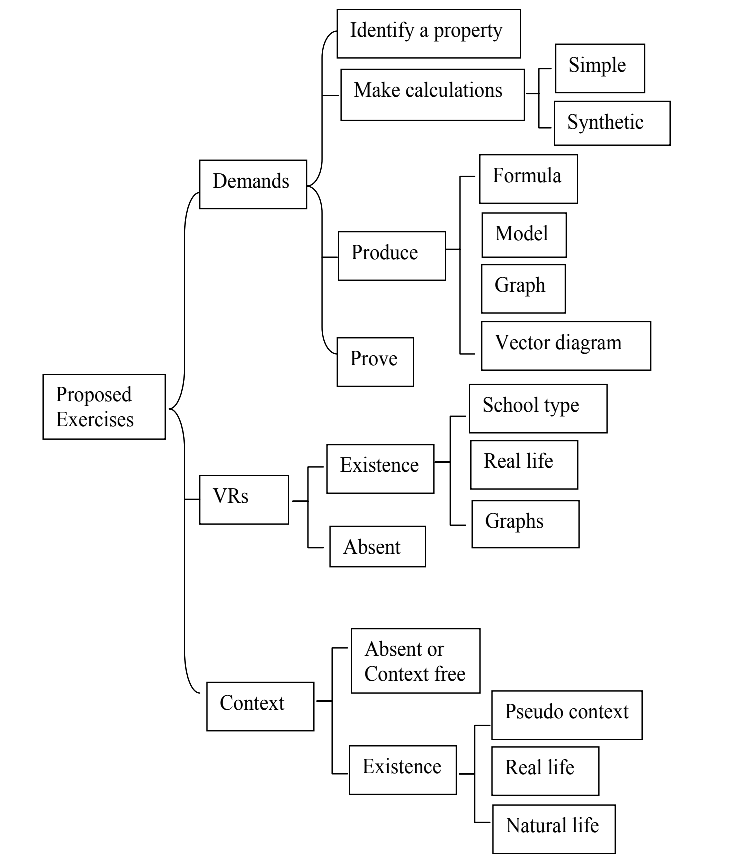


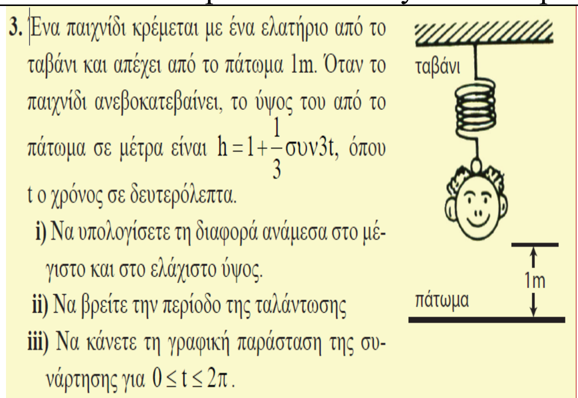
Figure 4.18: The systemic network on the dimension of the proposed exercises.

After analyzing a number of selected units in terms of the demands of exercises, eight further categories are defined: *identify a property*, *make simple* or *synthetic calculations*, *produce a formula* or a *graph*, or a *model* (a more demanding ask than producing a formula) or a *vector diagram* and finally *prove* a mathematical relation.

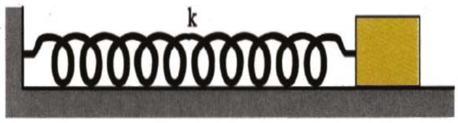
In terms on the VRs presented in the exercise two categories are defined: *absence* and *existence*. The second category was further analyzed according to the following subcategories: *School type* VR (we include schemes and figures), *real life* and *graphical representations*.

the analysis of the *contextual* aspects that supported the exercises, the following categories emerged: *context free* or absent, *pseudo-context* (where the exercises on the surface seemed to be about real world problems and situations, but actually had little connection to the real world), *real life* context and *natural* context.

In Table 4.16., we exemplify our analysis of the proposed exercises from the subject of mathematics, physics and applied technologies.

<table>
<tr><td colspan="2">Table 16: Examples of the analysis of the proposed exercises</td></tr>
<tr><td>
3. Ενα παιχνίδι κρέμεται με ένα ελατήριο από το ταβάνι και απέχει από το πάτωμα 1m. Όταν το παιχνίδι ανεβοκατεβαίνει, το ύψος του από το πάτωμα σε μέτρα είναι $h=1+\frac{1}{3}συν3t$, όπου t ο χρόνος σε δευτερόλεπτα.
i) Να υπολογίσετε τη διαφορά ανάμεσα στο μέγιστο και στο ελάχιστο ύψος.
ii) Να βρείτε την περίοδο της ταλάντωσης
iii) Να κάνετε τη γραφική παράσταση της συνάρτησης για $0 \le t \le 2\pi$.
</td>
<td>3. A toy hangs with a spring from the ceiling and is 1m from the floor. When the toy goes up and down, its height from the floor counted in metres is h=1+1/3sin3t, t is the time in seconds.
(i) Calculate the difference between the maximum and the minimum height (of the toy).
(ii) Evaluate the period of the oscillation.
(iii) Sketch the graph of the function for $0 \le t \le 2\pi$.</td></tr>
<tr><td colspan="2">Subject: Mathematics Grade 11
Ch. 3.4: Trigonometric functions

Analysis
Demands
Identify properties (i)
Make simple calculations (ii)
Produce a graph (iii)
VRs
Existence/school type
Context
Existence/pseudo-context</td></tr>
</table>

<table>
<tr>
<td>
Σώμα μάζας 0,2Kg ηρεμεί πάνω σε λείο οριζόντιο επίπεδο δεμένο στο ελεύθερο άκρο ελατηρίου σταθεράς 20N/m.
Αν το σώμα απομακρυνθεί λίγο από τη θέση του κατά τη διεύθυνση του άξονα του ελατηρίου και αφεθεί στη συνέχεια ελεύθερο:
α) να δείξετε ότι θα εκτελέσει Γ.Α.Τ.,
β) να βρείτε την περίοδό του.</td>
<td>A 0,2Kg body is in balance in a smooth horizontal surface tied from the free side of a spring having 20N/m constant.
If the body is removed towards the axis of the spring and is left free :
a. show that a Linear Harmonic Oscillation is going to be performed
b. find its period</td>
</tr>
<tr>
<td colspan="2">Subject: Physics Grade 11 Common core direction
Ch. 3. Trigonometry

Analysis
Demands
Prove (a)
Produce a formula (b)
VRs
Existence/school type
Context
Existence/pseudo-context</td>
</tr>
<tr>
<td>3⁰ Ωμική ανίσταση R = 40 (Ω) και πηνίο αυτεπαγωγής $L = \frac{\sqrt{3}}{10}$ (H), συνδέονται σε σειρά. Εάν η τάση στα άκρα του πηνίου είναι $v_L = 80\sqrt{3} \cdot ημ(400t + 90^\circ)$, να βρεθούν:
α) η στιγμιαία τάση τροφοδοσίας v
β) η στιγμιαία ένταση ρεύματος i
γ) η διαφορά φάσης μεταξύ ρεύματος και τάσης
δ) να γίνει διανυσματικό διάγραμμα τάσεων και ρεύματος
$(v = 160 \cdot ημ(400t + 60^\circ),\ i = 2 \cdot ημ\, 400t,\ φ = 60^\circ)$</td>
<td>Ohmic resistance R=40 Ω and spool $L = \frac{\sqrt{3}}{10} H$ are connected in series. If the Voltange in the end points of the spool is v=80 $\sqrt{3}$ sin(400t+90°) find:
a) The instant Voltage v
b) The instant intensity of current i
c) The phase difference between voltage and current
d) Sketch the vector diagram of voltage and current.</td>
</tr>
<tr>
<td colspan="2">Subject: Applied Technologies Grade 11
Specialization: Electronics
Chapter 9: Alternate Currents (AC)

Analysis
Demands
Produce a formula (a), (b) (c)
Produce a graph (d)
VRs
Non Existence
Context
CONTEXT FREE</td>
</tr>
</table>

4.5b) Quantitative analysis

4.5b1) The case on demands on exercises

Counting the frequencies of appearance of the final produced categories for the dimension "demands of the proposed exercises" involved in the textbooks of the three subjects gave us the results presented in Table 4.17 and in the Histogram, Figure 4.19.

Table 4.17: The relative frequencies in relation to the demands on exercises

Demands on exercises		MATHEMATICS No =85 (%)	PHYSICS No=58 (%)	APPLIED TECHNOLOGIES No=19 (%)
Identify a property (Prop)		0,21	0,19	0,32
Calculate	Simple (C1)	0,06	0,21	0
	Synthetic (C2)	0,15	0,16	0
	Formula (Form)	0,38	0,22	0,47
	Graph	0,16	0,02	0,17
	Model	0,07	0,38	0,11
	Vector diagram	0	0	0,16
Prove		0,18	0,09	0

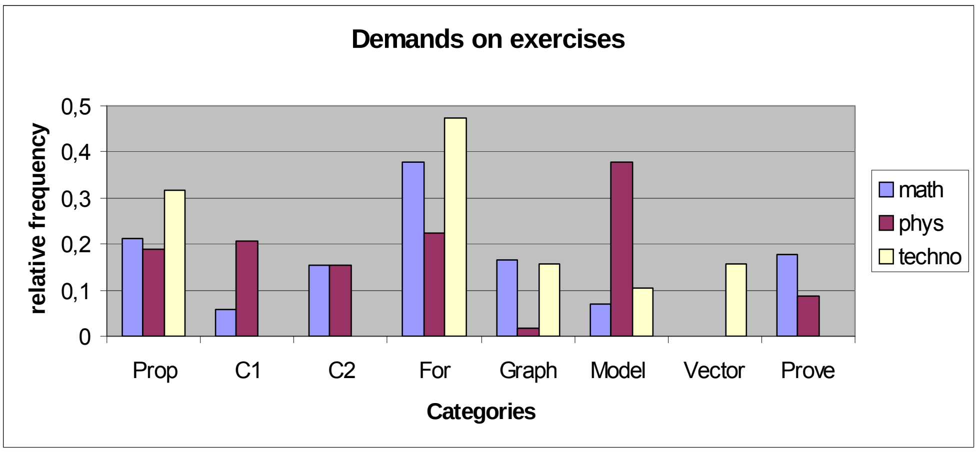


Figure 4.19: The figural representations of Table 4.17 data in the different subjects

Issues emerging

In general, it can be noticed the existed divergence in the demand of the proposed exercises analyzed in the three subjects.

Only in the category of identifying properties of periodicity we can see a similar approach in all subjects (20%-32%).

The subjects of mathematics and physics equally require from students to be involved in problem solving activities with synthetic calculations (15%).

Mathematics exercises mostly demand from the students to produce a formula (almost 40%) and in lower rates to prove a mathematical relation (20%) or produce a graphical representation (16%). Moreover, activities that demand from the students to produce a model are rather rare (7%).

Physics exercises demand mostly from the students to produce a model (almost 40%), than to prove (9%), which is more popular activity in mathematics (18%). Applied technology exercises demand mostly from the students to produce a formula (almost half of all the exercises in this subject falls in this category), while producing a graph seems to be an activity as important in this subject as in Mathematics. The case of demands on making simple or synthetic calculations seems to be absent in this subjects, while sketching vector diagrams with currents and voltages seems to be an important activity.

Overall, we have identified the differences met in the activities used in the textbooks for the practice and the development of new knowledge. This aspect is also very important for students' learning evaluation .

4.5b2) The case on VRs in exercises

Counting the frequencies of appearance of the final produced categories for the dimension "type of VRs" in the proposed exercises across the three subjects provided us with the results of Table 4.18. They are also presented inthe Histogram of Figure 4.20.

Table 4.18 The relative frequencies in relation to the VRs on exercises

VRs on exercises		MATHEMATICS No =85	PHYSICS No=58	APPLIED TECHNOLOGIES No=19
Non existence (abs)		0,82	0,65	0,89
Existence	School_type	0,14	0,07	0,05
	Real life	0,04	0,22	0
	Graph	0,01	0,02	0,05

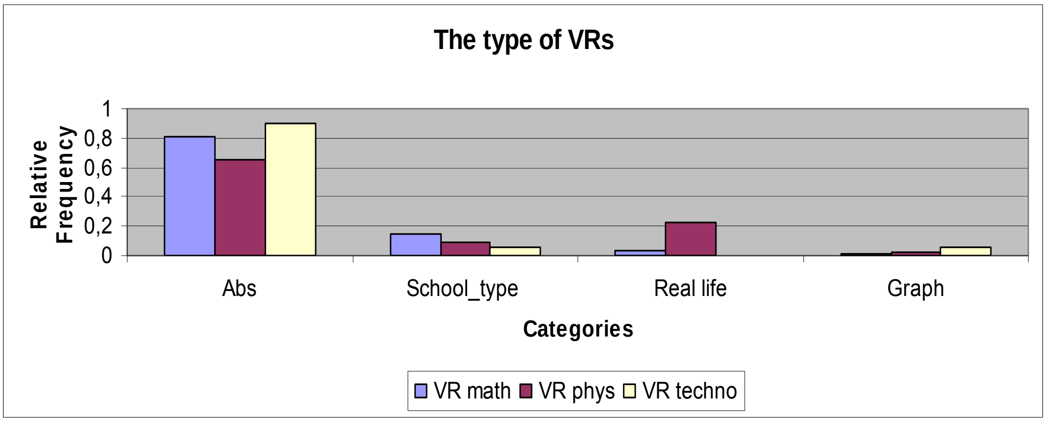


Figure 4.20: The figural representations of Table 4.18 data in the different subjects

Issues emerging

All subjects behave almost uniformly in terms of this category. VRs are almost absent in all exercises of all subjects. The only subject that uses images in the proposed exercises is physics. We can notice that 31% of all the exercises in physics are accompanied by a VR.

In the case where a VR exists in the proposed exercise, it is usually school type. Only in physics we meet in 22% of the proposed exercises VRs that represent real life activities.

It is interesting that very rarely graphical representations appear in the activities, although in the main texts graphical representations are used extensively.

4.5b3) The case of context in exercises

Counting the frequencies of appearance of the final produced categories for the dimension "type of VRs" in the proposed exercises in the textbooks across the three subjects resulted in Table 4.19 and in the Histogram of Figure 4.21.

Table 4.19: The relative frequencies in relation to the context in exercises

Context on exercises		MATHEMATICS No =85	PHYSICS No=58	APPLIED TECHNOLOGIES No=19
Non existence (Abs)		0,85	0,59	0,95
Existence	Pseudo-context	0,09	0,26	0,05
	Real life	0,01	0,09	0
	Nature	0	0,07	0

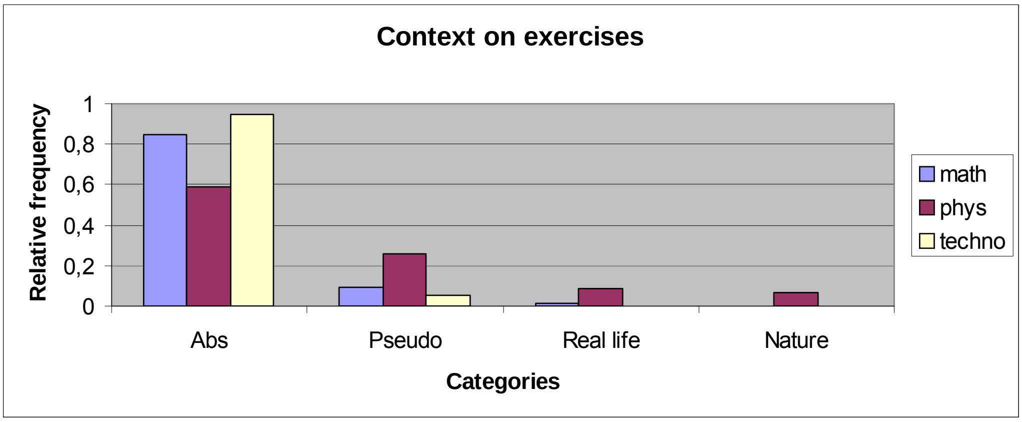


Figure 4.21: The figural representations of Table 4.19 data in the different subjects

Issues emerging

All subjects present almost uniform characteristics in this category. 95% of all exercises in applied technology, 85% of all exercises in mathematics, and 59% of all exercises in Physics are context free.

In the case that the exercises have a contextual meaning, this is usually a pseudo-context (posses only surface elements with the real world problems and situations). On the contrary, real life or nature contextual meaning is present only in physics with 9% and 7% rates respectively.

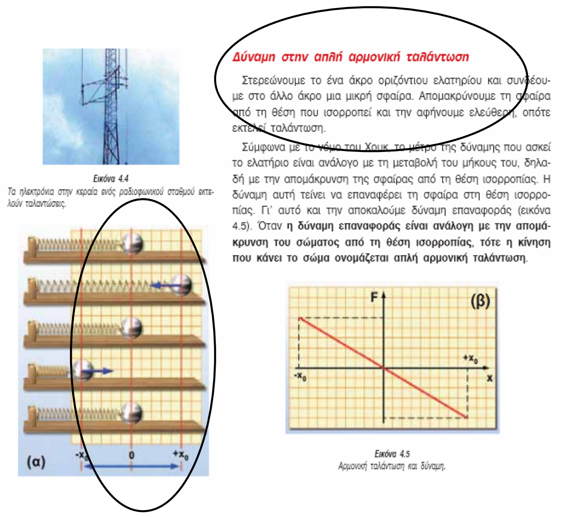
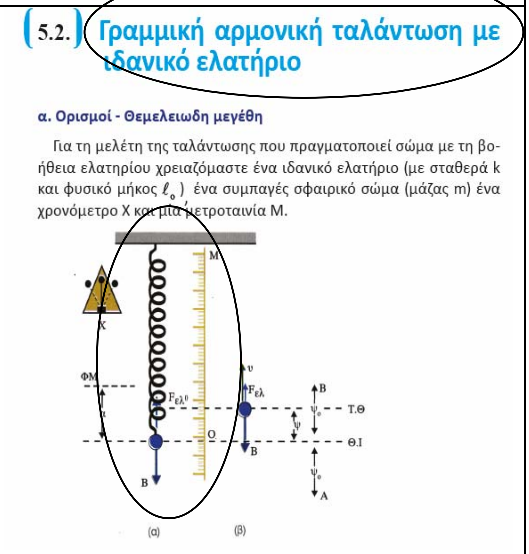
It is interesting that even in exercises in applied technologies authors avoid using exercises with real life meaning.

4.6) Other didactical issues concerning the Coherence across and in the subjects

4.6a) Linguistic issues

Naming specific type of oscillation (Physics)

One linguistic inconsistency that emerged from our analysis is the case of defining a specific type of oscillation in physics. Particularly, the same type of periodic motion (i.e. the periodic motion of an oscillating spring) is expressed differently in the textbooks in two different grades: In Grade 9 is defined as a “simple harmonic oscillation” while in Grade 11 as a “Linear harmonic oscillation”. In Table 4.20 we give evidence of this inconsistency.

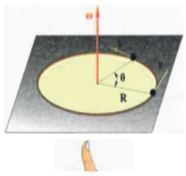
<table>
<tr><td colspan="2">Table 4.20: The two texts</td></tr>
<tr><td>Physics Grade 9 (Title: The force in simple harmonic oscillation)</td><td>Physics Grade 11a (Title: The linear harmonic oscillation with an ideal spring)</td></tr>
<tr><td>
</td><td>
</td></tr>
</table>

Using degrees and radians

In most applications of trigonometry, angles are measured in degrees. In more advanced work in mathematics, radian measure of angles is preferred. Radian measure allows us to treat the trigonometric functions as functions with domains of real numbers, rather than angles. A study that investigated pre-service and in-service mathematics teachers about their understanding of radians as the arguments of trigonometric functions concluded that most participants’ concept image of radians was not rich enough and was dominated by their concept image of degree (Topcu, Kertill & Akkos, Yilmaz, Onder, 2006).

In this direction, Thompson, Carlson & Silverman (2007) argue that textbook authors in U.S.A mathematics texts do not sufficiently help students to connect the two different trigonometries, the trigonometry of triangles and the trigonometry of periodic functions.

In Table 4.21 we present how a number of didactical issues could emerge on this issue in a mathematics and a physics text.

<table>
<tr><td colspan="2">Table 4.21: The mathematics (Grade 11) and the physics (Grade 11) texts</td></tr>
<tr><td>The original text</td><td>Translation</td></tr>
<tr><td>Πολλές εφαρμογές όμως των τριγωνομετρικών συναρτήσεων δεν περιέχουν γωνίες, αλλά πραγματικούς αριθμούς, όπως, π.χ, ο τύπος της αρμονικής ταλάντωσης $f(t)=\alpha\cdot ημωt$, στον οποίο τα α και ω είναι σταθερές και t είναι ένας πραγματικός αριθμός που παριστάνει το χρόνο.
Για το λόγο αυτό ορίζουμε στη συνέχεια τριγωνομετρικές συναρτήσεις πραγματικής μεταβλητής.</td><td>Many applications of the trigonometric numbers do not include angles, as for example the formula of harmonic oscillation f(t)=asinωt, where a, ω are constant and t is a real number that represents time.
For this reason we define trigonometric functions of real number.</td></tr>
<tr><td>Γωνιακή ταχύτητα στην ομαλή κυκλική κίνηση ενός κινητού, ονομάζουμε ένα διανυσματικό μέγεθος του οποίου:
· Η τιμή είναι ίση με το σταθερό πηλίκο της γωνίας θ που διαγράφηκε από την επιβατική ακτίνα σε χρονικό διάστημα t διά του αντίστοιχου χρονικού διαστήματος. Δηλαδή (Εικ. 8):
$$\omega=\frac{\theta}{t} \qquad (6)$$

</td><td>Angular velocity ω in the simple harmonic motion, is the vector quality that
Its value is equal the constant quotient of the angle θ that was traced by the passenger radius in the time interval t over this time interval. Hence:
$$\omega=\frac{\theta}{t}$$</td></tr>
<tr><td colspan="2">1st issue: f(t)=asinωt is not a trigonometric number but a trigonometric function
2nd issue: According to the extract from the physics textbook θ is an angle and ωt=θ, which contradicts the mathematics extract that states that in the symbolic formula f(t)=asinωt the argument of the function is not an angle.</td></tr>
</table>

The two texts are addressed to the same student who has to make the appropriate connections.

Mixing radians and degrees

In applied technologies texts many symbolic formula in the proposed exercises are mixing measures of an angle in degrees and radians e.g. $v_L = 80\sqrt{3}\cdot ημ(400t+90^{\circ})$, or $v_L = 80\sqrt{3}\sin(400t+90^{o})$.

4.6b) Epistemological issues

The definition of periodic function

Issues that emerging for different definitions of periodic functions in mathematics are already articulated in the article of Van Dormole and Zaslavsky (2003).

e.g. The definition of a periodic function in Mathematics exclude the function $f(t)=a\sin\omega t$, as a periodic function, since this function does not satisfy the necessary conditions to be periodical.

(A function f with domain the set A is called periodic, when there is a real number T > 0 so as for every $x \in A$: *i)* $x + T \in A$, $x - T \in A$ *and ii)* $f(x + T) = f(x - T) = f(x)$. *The real number T is called the period of f).*

The concept of periodicity

As Buendia and Cordero (2005) argue "the periodical property changes according to what is considered periodical in the particular field. In mathematics, in one hand, one talks about "periodic functions" and a function is considered periodical if it complies with the accepted definition. In the context of oscillators in physics, on the other hand, one speaks about "periodical phenomena" and here, ideas such as "almost periodical phenomena" are included" (p. 300).

This issue was identified in the Greek texts since in physics is not so clear that a damped oscillation does not represent a periodic motion since it does not satisfy the definition of a periodic motion.

Chapter 5: CONCLUDING REMARKS

The focus of this report is on analyzing the ways that the Greek school enculturate students in the conceptual field of periodicity in the subjects of Mathematics, Physics, Astronomy and Applied Technologies.

To this end, we analyzed 11 textbooks from the above subjects on selected units and specific topics related to the notion of periodicity. The textbook sample was from grade 9 in Lower secondary school, Grade 11 and Grade 12 in upper secondary school and Grade 11 in Technical and Vocational school in the specialities of Electronics, Electrology and Informatics. The total units analyzed were: 110 textual units, 214 visual representations and 162 proposed exercises.

The dimensions analyzed were (a) the conceptual aspects of periodicity appearing in textual units from all subjects; (b) the nature of argumentation developed and how periodicity is visually represented in the above texts and the co-deployment of visual representation and reasoning as well; (c) the proposed exercises; and finally (d) coherence issues across and in subjects.

Some concluding remarks from our qualitative and quantitative analysis are the following:

(a) The conceptual aspects of periodicity

Physics is the subject that seems to offer the richest learning environment, as different aspects of periodicity are introduced explicitly and are developed on the basis of every day and experimental experiences.

The most popular periodic motion in all subjects is the Rotational periodic motion, while the most popular function that models the periodic behaviour of almost all periodic motions is the sinusoidal function.

In both General and Vocational textbooks, Grade 11 is the level, where the main goal is every student to become aware and develop the notion of periodicity in a theoretical way.

(b) The nature of argumentation, the VRs and their co-deployment in all textual units

In order to implement our plan we developed a methodology of analyzing the argumentation developed in texts from different subjects. Our methodology defines two units of analysis: the argumentation developed in each conceptual thematic unit and the modes of reasoning the create the unit’s argumentation. In this way we can compare the argumentation, the generative activity and the tools mediating the argumentation in different texts in different grades and/or subjects.

The main finding of our analysis is the emergence of a scheme of categories concerning the modes of reasoning: Nomo-logical (N1, N2, N3, N4), Logical-mathematical, Logical-empirical (LE1, LE2 & LE3) and Empirical modes of reasoning (E1 &E2).

It has been found that all subjects use, in almost similar ways, the above main categories of modes of reasoning. However, the quantification of our data made apparent some interesting results:

The N3 (taxonomic category) is completely absent in mathematics. This may be an obstacle in students' knowledge since the categorization of different types of periodic motions or the functions that model their behavior is a demanding task.

As it was expected the category of Logical-mathematical reasoning is mostly present in mathematics; but also in physics this type of reasoning is used. The Logical-empirical (LE2) mode of reasoning is a unit, on the basis of which argumentation is built in all subjects. Finally, only in physics and astronomy reasoning it based on every day experiences (E1).

The presence of different routes in reasoning in different subjects that have the same goal (e.g. defining the period T by relying on previously established statements or by making genuine links to the real world) could motivate and allow students to broaden and enrich their perception of this notion.

Moreover, when we analyze the use of reasoning in argumentation across grades for the subject of physics we conclude: as we travel from the lower to the upper level, there is a tendency to move from empirical (empirical and logical-empirical modes of reasoning) to more abstract ways of thinking (logical-mathematical modes of reasoning); This is a consequence of the fact that in lower grades the argumentative activity is specifi and context-dependent, while in upper grades is context free and more abstract. This tendency is related to students' conceptualization and understanding and the teaching of periodic phenomena should take this into account. Furthermore, educating students to reason, while taking into consideration the contextual elements, could contribute to their development of active monitoring and self-regulating competences (Kaiser, 2009).

The exploration of the function of VRs in all textbooks reveals tools, practices and rules, used by the different communities showing the nature of activity taking place. The emphasis posed on abstracted aspects of periodicity in mathematical textbooks is apparent (95% of all visual representations in mathematics texts are free of any physical meaning) while physics is starting with the presentation of natural and everyday phenomena and follow a path up to semi-abstracted forms of knowledge (showing time-dependent images). So, mathematics texts on their own seem to fail in helping students to make connections between periodic phenomena of everyday life and/or the natural world and abstract mathematical notions. The other two subjects follow their historical goals (examples of Natural life are met mostly in Astronomy, while images of scientific devices are mostly met in Applied Technologies).

The above findings are also supported by analyzing the genre of VRs. Photographs are only included in science and astronomy textbooks, while all VRs in mathematical textbooks and applied technology are either graphs or schematic representations. Moreover, the main category of graphical representations in Mathematics, Physics and Applied Technology is that of sinusoidal functions. We have to mention that although the graphs of sine and cosine functions are sinusoids, the textbooks' authors prefer to use graphs of the sine rather than the cosine function in different cases.

The study of the co-deployment of visual representations and reasoning shows divergent behaviors in the different subjects. In mathematics texts VRs seem to undertake a significant role acting as fundamental tools of reasoning, while Astronomy and Applied Technology texts use images mostly as the starting point of the mode of reasoning. Finally, Physics texts are using VRs mostly as the basis of explanatory reasoning. These different ways of reasoning in relation to VRs may be not necessarily conflicting but complimentary for a scientist or a mathematician. Students, however, need to be supported in order to make the connections and fill in the existing gaps between the two activities in science and mathematics classrooms. This fact stresses educators' responsibility in being efficient to handle this task.

(c) The proposed exercises

In general, we have found that there is a divergence in the demand of the proposed exercises analyzed in the three subjects. Mathematics and Applied Technology exercises mostly require from the students to produce a formula. Physics exercises demand mostly from the students to produce a model. Also, producing a graph is a common demand in Mathematics and Applied Technologies, while VRs are almost absent in all exercises of all subjects. Moreover, almost all exercises in Applied Technology and in Mathematics textbooks are context free.

Overall, we can identify a divergence between the activities students involved when new knowledge is developed and when this knowledge is evaluating (i.e. the case of the proposed exercises). Specifically, the presence of context-dependent activities (e.g. logical-empirical or pure empirical modes of reasoning; 25% of all VRs in physics are possessing physical meaning based on every day examples; all subjects, except Astronomy, based their reasoning in sinusoidal graphs) are very common when new knowledge is developed. On the other side, context free activities (proposed exercises with no contextual meaning) are designed to evaluate this knowledge.

(d) Coherence issues

The study of linguistic practices showed aspects that may also affect students' learning. Characteristic inconsistencies emerging from our analysis are the cases of defining a specific type of oscillation in physics and also when arguing about moving from degrees to radians in mathematics. It seems that not enough attention is paid to the important question why we need radians instead of degrees in mathematics texts. All the above didactical issues (e.g. Linguistic inconsistencies among texts) and epistemological issues related to periodicity may be a source of possible students' misunderstandings.

The activity in the different scientific communities (Mathematics, Physics, Astronomy & Applied Technology) seem to have different goals, use different tools, and adopt different practices to accomplish them. The students throughout their education are expected to uncover all the above differences and consciously and intentionally construct links between them in order to develop a consistent understanding of the periodic phenomena. This is not always succeeded. The role of the educators is essential in order to integrate and incorporate the different aspects of periodicity addressed in the above subjects and support their students' conceptualization. It is suggested that teachers' knowledge and awareness of the characteristics of school textbooks may offer a new and deeper insight of what meaningful learning involves.

REFERENCES

Balacheff, N. (1988). Aspects of proof in pupils’ practice of school mathematics. In D.Pimm (Ed.), *Mathematics, teachers and children* (pp. 216-235). London: Hodder and Stoughton.

Biehler, R. (2005). Reconstruction of meaning as a didactical task: the concept of function as an example. In J. Kilpatrick, C. Hoyles, O. Skovsmose, P. Valero (Eds.) *Meaning in mathematics education*. Mathematics Educational Library V. 37; Springer. New York.

Bliss, J. Monk, M. & Ogborn, J. (1983). *Qualitative data analysis for educational research*. London: Croom Helm.

Boero, P.; Douek, N. & Ferrari, P. L. (2008). Developing mastery of natural language. In L. English (Ed.), *International Handbook of Research in Mathematics Education* (pp. 262-295). New York: Routledge.

Buendia G. & Cordero F. (2005). Prediction and the periodical aspect as generators of knowledge in a social practice framework. *Educational Studies in Mathematics 58*: 299– 333.

Cabassut, R. (2005) Argumentation and proof in examples taken from French and German textbooks. *Proceedings of 4th Cerme (congress of European society for research in mathematics education)*, Sant Feliu de Guixols.

Chi, M. T. H., de Leeuw, N., Chiu, M., & LaVancher, C. (1994). Eliciting self-explanations improves understanding. *Cognitive Science, 18,* 439-477.

Dreyfus T. & Eisenberg T. (1980). On teaching periodicity *International Journal in Mathematics. Education. Science & Technology*, 1980, Vol. 11(4), 507-509.

Frykholm,J. & Glasson, G. (2005). Connecting Science and Mathematics Instruction: Pedagogical Content Knowledge for Teachers. *School Science and Mathematics*, 105(3), 127-141.

Hanna, G. (2000). Proof, explanation and exploration: an overview. *Educational Studies in Mathematics*, 44, 5-23.

Kaiser, G. (2011). Mathematical thinking within mathematical modelling process. In Ubuz, B. (Ed.). *Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education*. Vol. I, pp. 91-96, Ankara: PME.

Kress, G & van Leeuwen, T. (2006). *Reading Images, the Grammar of Visual Design* (2nd ed.). London: Routledge.

Kuhn, T. (1962). The Structure of scientific revolutions**.** Chicago, IL: University Press

Leont’ev, A. N. (1978). *Activity, Consciousness, and Personality*. Englewood Cliffs: Prentice Hall

Love, E., & Pimm, D. (1996). 'This is so': A text on texts. In A. J. Bishop, K. Clements, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.), *International Handbook of Mathematics Education* (Vol. 1, pp. 371-409). Dordrecht: Kluwer Academic Publishers.

Mayring, P. (2000). Qualitative Content Analysis [28 paragraphs]. Forum Qualitative Sozialforschung / Forum: *Qualitative Social Research*, 1(2), Art. 20, http://nbn-resolving.de/urn:nbn:de:0114-fqs0002204.

Over, D. E., & Evans, J. St. B. T. (2003). The probability of conditionals: The psychological evidence. *Mind and Language, 18*, 340-358.

Roseman, J.E., Stern, L. & Koppal M. (2010). A Method for Analyzing the Coherence of High School Biology Textbooks. Journal of Research in Science Teaching, 47(1), 47-70.

Roth, W.M. & Lee, Y.J. (2007). "Vygotsky's Neglected Legacy": Cultural-Historical Activity Theory. *Review of Educational Research*, 77, 186-232.

Shama, G. (1998). Understanding periodicity as a process with a gestalt structure. *Educational Studies in Mathematics*, 35, 255-281.

Stacey, K. & Vincent, J. (2009). Modes of reasoning in explanations in Australian eighth-grade mathematics textbooks. *Educational Studies in Mathematics*, 72(3), p. 271-288.

Stinner, A. (1992). Science Textbooks and Science Teaching: From Logic to Evidence. Science Education, 76(1), 1-16.

Strauss, A. & Corbin, J. (1998). *Basics of qualitative research: Techniques and procedures for developing grounded theory* (2nd ed.). Thousand Oaks, CA: Sage.

Szu, E & Osborne, J. (2012). Scientific Reasoning and Argumentation from a Bayesian Perspective , *Perspectives on Scientific Argumentation*, Part 1, 55-71.

Tall, D. & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity, *Educational Studies in Mathematics*, 12(2), 151–169.

Topcu, Kertil, Akkos, Yilmaz & Onder, (2006). Pre-sesvive and in-service mathematics teachers' concept images of radian. . In Novotna, H. Moraova, H. Kratka, M. & Stehlikova, N. (Eds.) *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education.* (Vol.5, pp. 281 – 296). Prague: Charles University.

Thompson, P. W., Carlson, M. P., & Silverman, J. (2007). The design of tasks in support of teachers' development of coherent mathematical meanings. Journal of Mathematics Teacher Education, 10, 415 -432

Toulmin, S. (1969). *The uses or argument.* Cambridge, UK: Cambridge University Press.

Van Dormolen, J. & Zaslavsky, O. (2003). The many facets of a definition: the case of periodicity. *Journal of Mathematical Behavior*, *22*(1), 1-106.

von Bertalanffy, L. 1968. *General System Theory: Foundations, Developments, Applications.* New York: Braziller.

Vergnaud, G. (2009). The theory of conceptual fields. Human Development, 52, 83-94.

Vygotsky, L. (1981). "The Development of Higher Forms of Attention", in J. Werstsch (ed.), *The concept of Activity in Soviet Psychology*, 189-240. Armonk, NY: M. E. Sharpe.