Με τη συγχρηματοδότηση της Ελλάδας και της Ευρωπαϊκής Ένωσης

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Στοιχεία Μεταδιδάκτορος Ερευνήτριας: **ΤΡΙΑΝΤΑΦΥΛΛΟΥ- ΚΑΛΛΙΩΡΑ ΧΡΥΣΑΥΓΗ**

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FINAL REPORT ON DOCUMENTATION OF TEACHERS' THINKING

Ch. 1: Introduction

1.1 The overall aim of the research project

The present study is part of a research project that intends to take a close look at pedagogical practices adopted in mathematics and physics classrooms in Greek secondary schools on topics that are related to periodicity. Even though periodicity is central in a variety of disciplines, an extensive search of the literature shows that there are only a limited number of studies that focus on its understanding. These studies conclude that most students' conceived image of periodicity is based on time-dependent variations (Shama, 1998), while usually they consider any repetition as being periodical (Buendia & Cordero, 2005).

1.2 Summative results from the first and second phases of our research

To meet the aims of our inquiry, in the first phase of our project we analyzed Greek textbooks taken from the subjects of physics, mathematics, astronomy and applied technologies. Particularly, our analysis provided evidence about the reasoning and argumentation processes that the Greek mathematics and science textbooks adopted in the thematic units presenting the concept of periodicity and its properties. The results indicate that the above educational communities share traditional views on thinking as mental processes, while sensuous experiences are considered as less valuable in learning. Furthermore, in physics, functions such as $f(x)=e^{-bx}\sin(\omega x)$ that fluctuate in a periodical way on the x-axis, are considered as functions that model periodic motions. This disciplinary understanding of periodicity could encourage incorrect generalizations, such as, any type of repetition is periodical.

In the second phase, our main interest is how students perceive periodic motions and graphical representations. We take the position that understanding the notion of periodicity and its properties involves creating a coherent framework where ideas and educational practices in different school subjects are meaningful at an individual level. Further, understanding of periodicity is realized through specific situations where it takes its meaning (Radford, 2003). Therefore, we designed three different research activities (case studies) involving different aspects of the notion. The results indicate that (a) The vast majority of students easily identified the periodical property in periodic no sinusoidal graphs. It is interesting though that a graph, which exhibits a sinusoidal fluctuation (looks like the sinusoidal curve but with decreasing amplitude) seemed to confuse students a lot since almost seven out of ten considered that this represents a periodic motion. It seems that the distinction between sinusoidal and non-sinusoidal graph is not so clear, even though in mathematics and science, most students meet the sinusoidal function in their studies (as a trigonometric function or as the model of Simple Harmonic Oscillation; (b) Conceptualizing proportional relations of the quantities E (in Voltage) -υ (in m/s^2) on the formula $E=Bl\upsilon\sin a$ is more difficult than conceptualizing proportional relations of the quantities F-x on the formula $F=kx$. The second formula is the typical formula of examining linear relations in mathematics, while the first one is not; (c) students rarely make connections between different graphical representations of the same periodic phenomenon; (d) argumentation and reasoning on their claims seems to be a non-familiar practice for undergraduates students; (e) they exhibit strong willingness to assign meaning to abstract mathematical entities. The last finding shows the embodied nature of mathematical thinking and the genetic relationship between the sensual and the conceptual in knowledge formation (Nunez, 2007; Radford et al., 2004); Finally, (f) the formal mathematical tools, as the definition of periodic functions, do not seem to be enough to change such perceptions - even in the case of students who are studying mathematics.

1.3 The present report

Our aim in the present report

By taking into consideration the results of the two research phases, we planned the third phase with focus on educators' teaching practices and attitudes when they teach topics relative to periodicity in their classes. Particularly, we focus on their viewpoints about certain issues that arose during the first and second research phases and are seeking to discover if – and how educators in the various disciplines institutionalize their students' knowledge on aspects of periodicity. In order to investigate these issues we designed and conducted two research activities. Both were designed in a unified way for science, mathematics and engineering educators. In this manner, we expect to identify differences in educators' practices when they teach aspects of the notion of periodicity.

Our research activities and the relevant research questions are as follows:

Activity 1- questionnaires

We designed a questionnaire with open-end questions and placed emphasis on the role of visual representations (VRs) in educators' teaching practices when aspects of the notion are presented.

Our specific research questions are:

- Which images of periodicity they consider as fundamental in their teaching practices?
- How do they argue against students' misunderstanding of the periodic behaviour of two graphical representations?

Activity 2- interviews

We interview educators on the role of textbook argumentation in their teaching practice and ask them specifically about the role of the everyday examples they use when introducing aspects of periodicity:

- What is the role of everyday examples in their teaching practices?
- Do they follow the knowledge organization in texts in specific thematic unit related to the notion of periodicity?

In both activities, we ask the educators their opinion on how to contribute to their students' development of a unified way of understanding periodicity.

Outline of this report

We present the theoretical framework chapter 2 and in chapter 3, examine the methodology applied. In Chapter 4, we highlight the results of the qualitative and quantitative analysis that emerged from this analysis. Finally, in Chapter 5 we present the general conclusions and issues we want to research in depth in the next phases.

Ch. 2: Theoretical framework

In this study, we adopt a socio-cultural perspective where knowledge (about worldly objects and events) and linguistic knowledge (about sign forms) are mediated in and through the activities an individual engages in (Vygotsky, 1978). According to this point of view, thinking about physical phenomena enriches and promotes the development of students' scientific and mathematical knowledge (Buendia & Cordero, 2005).

Particularly, we adopt the activity theory perspective (ibid.) that recognizes mathematical and scientific school practices as different cultural activities, since they have different goals, purposes and objectives. In general, mathematics deals with patterns and relationships (NCTM, 2000), while science deals with the understanding of everyday and natural world phenomena (NRC, 1996). Despite these differences, a consensus holds among the above disciplines that instruction should help students to gain understanding of how scientific/mathematical claims can be proved or disproved (Oehrtman & Lawson, 2008); to develop and evaluate mathematical arguments; to select and use various types of reasoning and proof (Stylianides, 2008; 2009; Stylianides & Stylianedes, 2009). Moreover, Mahidi (2013) by analyzing the knowledge organization in both university-level physics textbooks and teachers' perceptions identified inductive-like and deductive-like structures.

By taking into consideration the above issues in the present study, we aim to explore how educators utilize the argumentation developed in the textbooks (as the major didactic resources) on specific thematic units that present aspects of the notion (Activity 2- interviews).

Our epistemological view on teaching is that it consists of generating and keeping in movement contextual activities which are situated in space and time and heading towards a fixed pattern of reflective activity incrusted in the different school cultures (in our case mathematics, science and engineering). This movement has three essential characteristics: (a) the object (that, in our case, is the notion of periodicity) is not a monolithic or homogenous object. It is an object made up of layers of generality and these layers will be more or less general depending on the characteristics of the cultural meanings of the fixed pattern of activity in question. An example of this is the kinaesthetic movement of a child that plays in a playground swing by forming a periodic motion in a certain time interval and the graphical representation of the above movement as height-time variation. The layers of generality are noticed in a progressive way by the student. The learning process consists of finding out how to take note of, or how to perceive these layers of generality (Radford, 2013).

Teaching aspects of periodicity in mathematics and science involves images of instances (or aspects or properties or models) of the notion. These representations in a school text are expressed either visually (e.g. pictures, diagrams or maps) or symbolically (e.g. equations or formulae). The role of images of a common notion in different teaching practices remains under investigated. We consider that the representations of the notion of periodicity are cultural resources which act as bearers of distributed intelligence (Pea, 1993) and that they carry, in a compressed way, socio-historical experiences of cognitive activity and artistic and scientific standards of inquiry (Lektorsky, 1995). These ubiquitous mediating structures both organize and constrain educators' teaching practice and provide to students a specific, conceptually structured space to think (Radford, 2013). Are the educators in the different disciplines adequately equipped to handle the above issues? This is another aspect we try to investigate in this study (Activity 1-questionnaires).

Ch. 3: Methodology

We planned the two research activities to be based on open-end questions. The data analysis was based on the grounded theory research perspective (Corbin & Strauss, 2007). In particular, we are looking for categories and patterns emerging from the analysis of the raw data. More specifically, inductive content analysis (Mayring, 2000) was applied on specific thematic units and a coding system of categories has been produced.

3.1 The participants

A total of 49 educators (41 – and 13 in research activities 1 and 2 respectively while 5 of them participated in both activities) participated in this research phase. It should be noted that although we distributed 84 hard copies of the questionnaire and sent 75 electronically, we only received 41 completed questionnaires (24 in hard copy and 17 electronically). Educators' reluctance to participate in this study could be due to the type of questionnaire used (open-ended tasks) or for reasons not relevant to this study (e.g., due to the financial crisis, many educators feel uncertain about their jobs, especially since 2000 educators in vocational schools have lost their jobs last year.

Activity	Engineering	Mathematics	Physics	TOTAL
1	6	20	15	41
2	3	5	5	13

3.2 The research activities

The research activities are based on some of the results from first and second phase research.

Activity 1-questionnaires

A questionnaire with open-end questions was either distributed as hard copy to some educators in 13 schools in three Greek cities, or sent electronically to others. In this activity our focus is on visual representations of the notion that we met in school texts.

The questionnaire consisted of four tasks

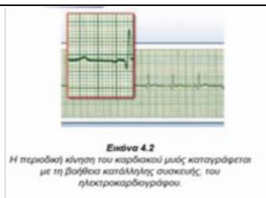
Task 1

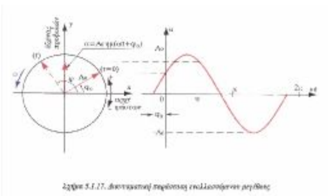
We ask the educators to refer to the basic teaching units they teach that contain aspects of the notion of periodicity, choose one teaching unit of the above and make a list of the fundamental visual representations (VRs) of the notion that they meet in the school textbook in the specific teaching unit and finally, choose one of the visual representations mentioned above (in question 2) and describe how it is used (its role) in the textbook in the specific teaching unit.

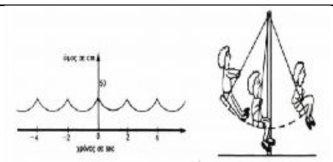
Task 2

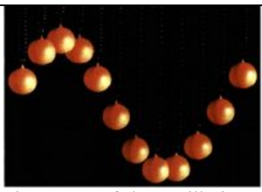
We selected five images (and the accompanying captions) of the notion of periodicity, as we found them in five different school texts. All images represent graphs of periodic motions in the different cultures. First of all, the selected images function differently in the reasoning developed in the different school textbooks (VR1 as an everyday example of a periodic motion, VR2 & VR3 play a fundamental role in reasoning in engineering and maths respectively, while VR4 & VR5 are used as complementary tools in the argumentation developed. Furthermore, the last two representations are images of the same physical phenomenon, the oscillation of a spring that is expressed in different contexts. VR4 is used in the context of physics (a ball that oscillates on an ideal spring) while VR5 concerns the engineering context (the body of a car that oscillates with the car suspension system before applying the absorber. Secondly, the graphical images are sinusoidal (VR2, VR4 & VR5 are the basic models of periodicity in all school cultures while VR1 & VR3 represent periodic but not sinusoidal graphs. Thirdly, the genres of each representation is different (VR1 is a photo, VR4 an elaborated photography, VR3 & VR5 are a combination of drawings and sinusoidal graphs and VR2 is a combination of a schematic representation (the trigonometric circle) and a sinusoidal graph.

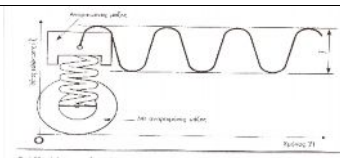
The five VRs and their theme are presented below.

VR1	VR1's theme
 **Physics, 3rd year in lower secondary school**	It represents a contextual application of a periodic motion (the motion of the cardiovascular muscle).

VR2	VR2's theme
 **Electrotechnology, 1st Grade, 1st Circle, Technical and vocational school**	It represents the clockwise circular motion of α vector representing the alternate current $a(t) = Ao \sin(\omega t+\varphi)$. The angular velocity of the above motion is ω. The starting point of the motion (t=0) was ($Ao\sin(\varphi o)$). Next to this motion is its graphical representation.

VR3	VR3's theme
 **Algebra, 2nd grade Upper secondary school, general Direction**	It represents a contextual application of a periodic motion (the kinesthetic movement of a girl on a swing) and the graphical variation of the height with respect to t.

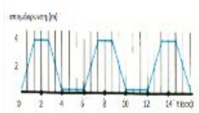
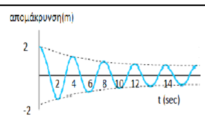
VR4	VR4's theme
 Ph. 1.2 Consecutive instances of the oscillating sphere attached on an ideal spring. The interval between two shots is always the same. During the photo shoot, the photo block moves horizontally with constant speed. In this way the photo shows how the vertical displacement changes with respect to time. **Physics, 3rd grade upper secondary school, Scientific direction**	It represents consecutive instances of the oscillating sphere attached on an ideal spring.

VR5	VR5's theme
 Fig. 4.29: Oscillation of a spring and the car body without using an absorber **Car systems, I, 2nd year in 1st circle, technical & vocational school**	The oscillation of the body of the car due to car suspension system in the case that the absorber is not used.

We consider that all images have the potential to mediate teaching practices in all disciplines. In this task, we question the acceptance or denial of the actual or the potential use of each particular VR. Only in the case of using the particular VR in their classroom did we ask the educators to indicate their actual or potential teaching goal.

Task 3

We use results taken from research phase 2. In the following task, we asked the educators to describe in brief how they could handle three characteristic students' responses (in cases when these came up in their classes).

Students' Task		
Graph 1	*Graph 2*	***Questions***
		*Does this graph represent a periodic motion? Justify your answer.*

Students' responses
R1: Graph 1:«It is not periodic because the function (represented) is not sinusoidal»
R2: Graph 2: «It is periodic because it represents the motion of the swing»
R3: Graph 2: «It is periodic because every sinusoidal function is periodic»

We chose the above students' responses because they all refer to the sinusoidal function either explicitly (R1 & R3) or implicitly (R2). Besides, in the case of graph 2, theses responses were typical and seemed to predominate students' conception that a function that fluctuates about the x-axis with a decreasing amplitude is periodic. In the case of graph 1, the small percentage of students who replied that the graph does not represent a periodic motion, justified their response in this way.

In this task, we questioned educators' ways of handling the above responses in their classes.

Task 4

We asked the educators to write down their suggestions on how they could help their students to develop a unified view on periodicity where aspects of the notion from the different subjects coexist harmonically.

Activity 2 - interviews

The data source in this activity includes 13 individual semi-structured interviews. The interview themes were based on the role of textbook argumentation when topics on aspects of periodicity are presented.

Some of the themes we discussed with the educators were:

(1) **The role of everyday examples** when introducing the notion of periodicity. Some questions were: Do you use in your teaching practice the examples provided in the textbooks or other examples? Can you specify? How do you connect the examples with the topic you teach?

(2) **The utilization of the argumentation developed in school textbooks** on specific topics when the new knowledge is presented. Particularly, we chose specific topics on periodicity from each subject and discussed how the educator presents and supports this topic to their students. The topics are:

- Periodic functions and Studying the sinusoidal function (Math, 2nd year in upper secondary school);
- Periodic motions (Physics, 3rd year in lower secondary school); and Define linear harmonic oscillation (Physics, 2nd year in upper secondary school);
- The teachers in engineering courses chose the topic themselves.

Some questions were: *In the school text the new knowledge is developed in a certain way. Do you follow this when you teach one of the above topics*? *Are there some parts on the development of the new knowledge that you pay more attention to when you teach this topic*? *If yes, which ones exactly*?

(3) We asked educators for suggestions that could help the students develop a unified view on periodicity where aspects of the notion from the different subjects coexist in a harmonic way (The same with Activity 1, Task 4).

3.3. Data analysis

The data were educators’ written or electronic responses in Activity 1 and the analysis of the audio-recorded and hence transcribed data taken from Activity 2. Inductive content analysis (Mayring, 2000) was applied on educators’ responses in all activities and tasks.

Ch. 4: Results

We present below the results of the two research activities on each task and each question separately.

4.1 Questionnaire - task 1

(a) The main teaching units educators mentioned in which they teach aspects of the notion of periodicity is presented according to subject in Table 4.1

TABLE 4.1: The main teaching units

Subjects	*Teaching units where aspects of periodicity are present*
Engineering	alternate currents, signals
Mathematics	Periodic functions, trigonometric functions, the circular motion of the earth around the sun (astronomy), the decimal expansion of a rational number with a recurring, non-terminating part.
Physics	Oscillations, waves, alternate currents, circular motions, signals

The teaching unit they chose to exemplify, and the fundamental visual representations in the relevant unit are presented in Table 4.2

TABLE 4.2: The fundamental images of periodicity in school texts*

Educators' subject	*Teaching unit*	*Fundamental visual representations in school texts*
Engineering	alternate currents	generator of alternate current and the corresponding sinusoidal function, photo of moto-electrical devices
Mathematics	trigonometry	sinusoidal graphs, other graphs of trigonometric functions, the trigonometric circle, drawing of playground swing
Physics	Harmonic oscillations	Sinusoidal graphs, graphs of linear relations, the photo of the pendulum clock, photo or drawings of the playground swing, schematic representations of experiments with springs and simple pendulums

*Some of the educators added the use of digital technology (videos, animations or simulations).

According to our data, the most fundamental images of the notion in school texts are:

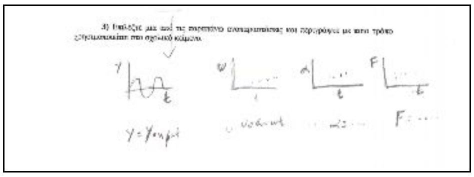


Fig. 4.1: Drawings in Q38_physics

(a) the *sinusoidal curve* which seems to dominate educators' practices in all subjects. The educators use this curve in different contexts and for various purposes (i.e. in the symbolic form of $I = I_o\sin(\omega t)$ to study alternate currents in engineering courses, in the symbolic form of $y = \sin x$ to study trigonometric functions in mathematics and in the symbolic form of $y = A\sin(\omega t)$) to study simple harmonic oscillations in physics. A characteristic response is presented in Fig. 4.1 where the educator draws a collection of variations (y-t, u-t, a-t and F-t) that are modeled by sinusoidal functions.

(b) The second image is the *pendulum swing*. This image appears in different genres and in different contexts and seems to be a common image of the notion in mathematics and physics texts. Particularly, the swing of the pendulum image appears as a photo of a pendulum clock in physics or as a drawing of a playground swing in physics and mathematics or as a schematic representation of a simple pendulum swing (a pendulum swing consists of a relatively massive object hung by a string from a fixed support) in physics.

The rest of the images the educators referred to seem to accommodate the needs of each subject. For example, there are photos or drawings of the generator of alternate current or moto-electrical devices in engineering texts, and the schematic representations either of the trigonometric circle in mathematics or of experiments with springs in physics texts.

In the question of choosing one of the visual representations mentioned above and describe how it is used (its role) in the textbook in the specific teaching unit, we have the following results.

TABLE 4.3: VR's role in textbooks/educators' responses

Type of VR	*VRs' role in textbook*	*Educators' subject*
Trig. circle	To study the variation of the sinx function	Mathematics
The pendulum swing	To visualize the periodical change of quantities in the course of time	Mathematics & physics
The drawing of the playground swing	Define the periodic function	Mathematics
Sinusoidal functions	To solve trigonometric equations, study the characteristics of the function	Mathematics
	To model the periodical change of quantities in the course of time	Physics & engineering
Synthesis of drawings, graph & photos (e.g., moto-electrical parts)	Explains the principles of operations of technological devices	Engineering
General	To attract students' attention	Physics, mathematics & engineering
	To visualize the real situation	Physics, mathematics & engineering
	To generalize and define	Mathematics
	Relate the math model and the physical phenomenon (when the image consists of a synthesis of a physical situation and the mathematical situation that models the first one)	Mathematics & physics

As in Table 4.2, the pendulum and the sinusoidal function seem to be recognized by the educators as the typical images of periodicity in school texts. According to educators' responses, the role of these images is not always the same. For example, the sinusoidal curve is used in physics and engineering texts to model the periodical change of quantities in the course of time, while in mathematics texts, its purpose is to solve trigonometric equations or study the characteristics of the function (e.g., max, min, etc.). This is also the same with images that represent '*the pendulum swing*'. These images' roles in texts are considered by the educators as dissimilar. The reason for this dissimilarity seems to be the layer of generality of the above object. In the abstract form (simple pendulum in physics), its role is to visualize the periodical change of quantities in the course of time, while in its concrete form (the drawing of the playground swing) it is to define periodic functions. It seems that the educators have varying opinions about how the same image/content plays a role in the texts or what purpose they are used for.

Many educators from all subjects consider that the role of a visual representation in a text could be either to attract students' attention or to visualize the real situation. Educators in engineering courses consider as important synthetic images consisting of different types of VRs. As an example, we provide the following image that is considered as important in Q25_eng. This image is a synthesis of a graph (the sinusoidal curve), a drawing (generator of an alternate current) and two photos (the alternator). This synthetic image role in an engineering text (Car electrical system) is to explain the operation of this moto-electrical device (in this case, the alternator). This image incorporates elements of mathematics, physics and technology.

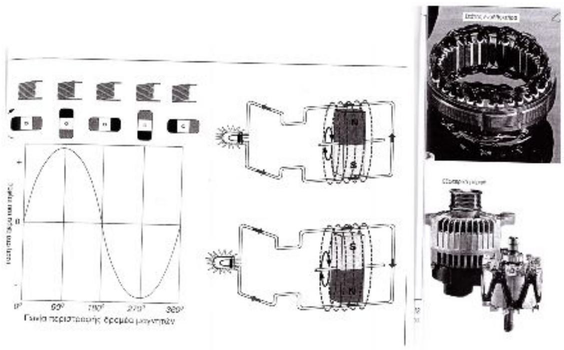
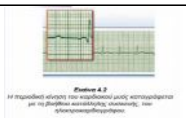
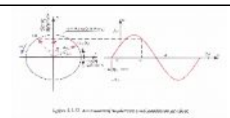
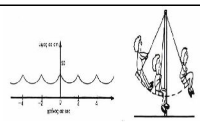
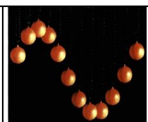
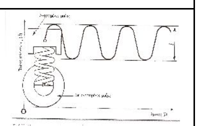


Fig. 4.2: The VRs' mentioned by an engineering educator (Q25_eng)

Questionnaire - Task 2

In this task, we question the acceptance or denial of the actual or the potential use of five VRs selected from five different school texts. Only in the case of using the particular VR in their classroom, did we ask the educators to indicate their actual or potential teaching goal.

<table>
<tr><th colspan="5">TABLE 4.4: The five images of periodicity (VRs)</th></tr>
<tr><td>
VR1</td><td>
VR2</td><td>
VR3</td><td>
VR4</td><td>
VR5</td></tr>
</table>

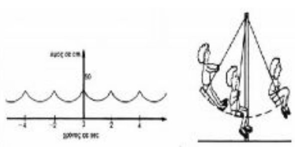
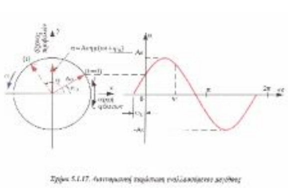
On the task of choosing particular VRs for actual and potential use, the educators' responded as follows:

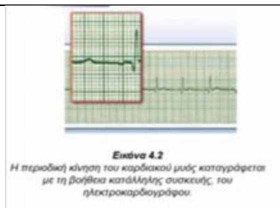
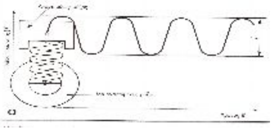
The most preferable VR by all educators was VR3 (the synthesis of the drawing of a playground swing and the periodic graph). The second most preferable VR by all educators was VR2, in the third place we meet VR4. The respondents put VR1 in fourth place and VR5 last.

TABLE 4.4: Educators' hierarchical preference of the five VRs	
VRs	*Actual & potential use* *41 participants*
VR1	19
VR2	27
VR3	32
VR4	23
VR5	9

It seems that although the majority of the educators actually and potentially will prefer images abstract (VR2) or semi-abstract (VR3) the concrete image in the chronophotography (VR4) is appreciated as teaching tool as well. On the other side, the semi-abstract image of the car vibration and the concrete image of the heart muscle motion seem to be not preferable images.

In Table 4.5 we present educators' preference on particular VRs for actual or potential teaching from most popular to less popular image, and their a priori-analysis.

TABLE 4.5: Educators' actual and potential use of the five VRs and VRs' a priori analysis		
The use of the VRs	*The VRs*	*VR's a priori analysis*
Actual use (N= 21) Eng: 0; Math: 20; Physics: 1. Potential use (N=11) Eng: 0; Math: 1; Physics: 10.	**VR3** 	***Context*** *Physical situation:* represents the kinesthetic motion of the girl's movement on a playground swing. *Mathematical situation:* The graphical representation of the height-time variation. *Layer of generality*: Two layers (concrete) - semi-abstract (represents the variation height (t) that is repeated in every 2 seconds. ***Genre*** A synthesis of a drawing and a graph of a periodic function (non-sinusoidal).
Actual use (N = 19) Eng: 3; Math: 8; Physics: 7. Potential use (N = 8) Eng: 1; Math: 3; Physics: 4	**VR2** 	***Context*** *Physical situation:* None. *Mathematical situation:* vector representations (of the alternate current), the sinusoidal graph as the basic model of periodic variation, the trigonometric circle as the basic model of periodical behaviour and symbolic entities ($a=Ao \sin(\omega t+\varphi)$) *Layer of generality*: abstract *Genre:* Synthesis of a schematic representation (the trigonometric circle) & a periodic graph (the sinusoidal graph).

Actual use (N = 12) Eng: 1; Math: 0; Physics: 11 Potential use (N = 11) Eng: 2; Math: 7; Physics: 2.	**VR4** 	***Context*** *Physical situation:* Instances of the vertical displacement of a sphere attached to a spring (the phototrophic image is elaborated). *Mathematical situation:* The formation of the sinusoidal function *Layer of generality*: concrete *Genre:* Photo – chronophotography
Actual use (N = 9); Eng: 0; Math: 0; Physics: 9. Potential use (N = 10) Eng: 1; Math: 8; Physics: 1.	 **VR1**	***Context*** *Physical situation:* the kinesthetic motion of the cardiovascular muscle. *Mathematical situation:* Graphical representation of the physical situation (x & y axis are not specified). *Layer of generality*: Concrete ***Genre*** Photo of everyday situation.
Actual use (N = 2) Eng: 2; Math: 0; Physics: 0. Potential use (N = 7) Eng: 0; Math: 4; Physics: 3.	 **VR5**	***Context*** *Physical situation:* The vertical displacement of a car body with respect to time. *Mathematical situation:* The formation of the sinusoidal function as the displacement-time variation. *Layer of generality*: a synthesis of concrete (the car body) and semi-abstract (the sinusoidal graph) ***Genre*** A synthesis of a drawing and a periodic graph (sinusoidal)

The results indicate that the synthetic image of a playground swing and a periodic graph dominate (actually and potentially) mathematics and physics teachers' practices. This result adds that the 'pendulum swing' is a prototypical image of periodicity in the above subjects. Some math educators admitted, though, that their reference to this image is very brief due to time restrictions. Some physics educators suggested changing parts of this image for potential use in their class. The suggested changes by two participants were to put the starting point of the motion at (0,0) while another three acknowledged that the period of the motion is different in physics, and this must be mentioned to students. This critical perception of an image indicates that educators many times are able to change something given in their text in order to use it as a tool in their teaching practice.

The second most preferable image was VR2 that represents the clockwise circular motion of α vector representing the alternate current $a(t) = Ao \sin(\omega t+\varphi)$. The angular velocity of the above motion is ω. The starting point of the motion (t=0) was ($A_o\sin(\varphi_o)$). Next to this motion is its graphical representation. This image layer of generality is considered abstract, as no physical situation is represented while a variety of mathematical objects are present (the trigonometric circle, the vector representations of the alternate current), the sinusoidal graph as the basic model of periodic variation and many symbolic entities ($a=A_o\sin(\omega t+\varphi)$). This image's potential use seems to dominate physics and engineering teachers' practices, while only a few math educators mentioned its potential use in their class. The issue that this image reasons visually on the generation of the sinusoidal function, but in the context of alternate current, seems to be restricted by most of the math educators (maybe they were not familiar with many symbolic entities in this VR).

The third place is VR4. It is an elaborated photo (chronophotography) and its layer of generality is concrete. VR4 represents consecutive instances of the oscillating sphere attached on an ideal spring. In the fourth place we find VR1 that represents the motion of the cardiovascular muscle.

In the last place we find VR5, which represents the periodical vibrations of the body of the car due to vibrations of a spring suspension system in case the absorber is not used. Although the spring periodic motion is in the curriculum of physics courses, only a few physics educators mention that they could use it in their class.

On the task of writing down their actual or potential teaching goals, the categories that emerged are presented below:

- to make connections to everyday phenomena;
- to study the periodic motion exhibited in the image (e.g., max, min, width, frequencies, rotating vectors);
- to study the represented mathematical model (e.g., specific instances of the variation on x & y axis, phase shifts, calculate the period or the frequency of the motion);
- to relate the periodic motions with their mathematical models
- to define certain aspects of periodicity (e.g., periodic functions)
- to relate different models of periodicity (e.g., the trigonometric circle and the sinusoidal function).

In the following table we present the frequency of the most expressed teaching goals in educators' responses for all VRs in both the actual and potential uses.

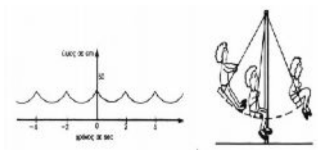
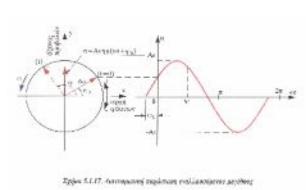
TABLE 4.6: Educators' teaching goals when using a particular VR in an hierarchical order

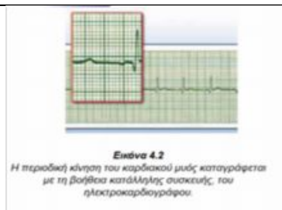
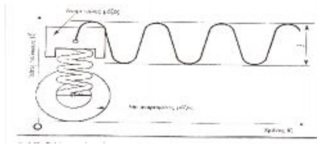
Categories emerged	*Engineering N=6*		*Mathematics N=20*		*Physics N=15*		*Total N=41*
	Actual	*Potential*	*Actual*	*Potential*	*Actual*	*Potential*	
Study the represented mathematical model	1	0	9	6	9	5	30
Relate periodic phenomena & mathematical models	1	2	8	3	5	6	25
Make connections to everyday	1	1	2	8	7	4	23

phenomena							
Relate different graphical images of periodicity	1	0	2	0	3	0	6
Define certain aspects of periodicity	0	0	2	0	2	0	4
Study the periodic motion exhibited in the image	0	1	1	0	0	1	3

In actual and potential use the most mentioned teaching goals were (a) to study the represented mathematical model; (b) Relate periodic phenomena & mathematical models; and (c) to make connections to everyday life phenomena.

In the following table, we present the most expressed teaching goals by educators in the different subject for each VR. In the last column, we present the role of each VR in the reasoning developed in the particular school text (results from the first part of the current research project).

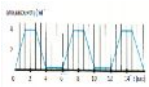
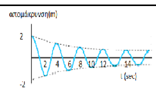
TABLE 4.7: The expressed teaching goals by educators for each VR		
The expressed teaching goals by educators for each VR	*VRs*	*The role of the VR in argumentation developed in the school text*
In actual use and potential use: all categories of teaching goals	**VR3** 	Fundamental role in defining periodic function (mathematics text)
In actual use and potential use we meet only the following three categories: (a) to study the represented mathematical model; (b) to relate the periodic motions with their mathematical models; (c) to relate different models of periodicity	**VR2** 	Fundamental role in defining the alternate current (electro technology text).
In actual use and potential use: all categories of teaching goals	**VR4** 	A complimentary role in defining simple harmonic oscillation in the case of an ideal spring (physics text).

In actual use and potential use we meet mostly the category: to make connections to everyday phenomena (all educators).	 **VR1**	On exemplifying on the case of periodic motions (physics text)
In actual use and potential use: most of the categories of teaching goals	**VR5** 	Illustrating the car vibrations on car suspension system (Car system text)

The above results indicate that the same image could be adopted in classroom activities in divergent ways. Only in the case of VR1, the photo of the motion of the cardiovascular muscle, there was a consensus among all educators when expressing actual and potential uses in their classes The rest of the images were placed under different goals mostly to make connections to everyday phenomena, while the same image in its actual use is to study the represented mathematical model.

Questionnaire, Task 3

In this task we use results taken from research phase 2. Particularly, we asked the educators to describe in brief how they could handle three characteristic students' responses (in cases when these came up in their classes) in the following task:

Students' Task		
Graph 1	*Graph 2*	*Questions*
		*Does this graph represent a periodic motion? Justify your answer.*
The students' responses		
R1: Graph 1: 'It is not periodic because the function (represented) is not sinusoidal'		
R2: Graph 2: 'It is periodic because it represents the motion of the swing.'		
R3: Graph 2: 'It is periodic because every sinusoidal function is periodic.'		

The analysis of educators' responses on this task showed that two main categories emerged: The type of reasoning educators adopted when they were commenting on the students' answers, and the type of communication they suggested.

In the category of *type of reasoning* adopted by the educators the subcategories that emerged were:

- Compare the empirical (concrete) situation with the represented mathematical model. Some educators express their concerns about this type of comparisons e.g., "*we can not define the notion periodicity through an example*" (q7_math);
- Use nomo-logical modes of reasoning (by applying the definition, refer to taxonomies of periodic motions, make clear that not any function that

contains sinx is periodical etc.). In this case, the concrete situation is absent;

- Compare same type of representations (either mathematical models or concrete situations) (e.g., educators are looking for similarities and differences between periodic graphs).
- Judging students' response as correct or incorrect with no further comments

In the category *type of communication,* we acknowledge two main subcategories educators suggested

- a discussion with the students (educator suggested starting a discussion about issues that had emerged)
- Monologue (no trace of students' presence in educators' response).

In the category *type of reasoning* used by educators when handling the particular responses (R1, R2 & R3), the following results emerged:

TABLE 4.8: Educators' reasoning on students' responses

The categories	*R1*	*R2*	*R3*
Use nomo-logical modes of reasoning (by applying the definition, taxonomies of periodic motions, etc.).	17	12	13
Compare same type of representations (either mathematical models or concrete situations)	5	2	6
Compare the empirical (concrete) situation with the represented mathematical model	2	9	0
Judging students' response as correct or incorrect with no further comments.	0	0	4

In all three responses (R1, R2, R3) educators seemed to prefer '*Use nomo-logical modes of reasoning'* when presenting how they would handle the above issues in their classes. In the second place, we find the category of '*Comparing same type of representations (either mathematical models or concrete situations)*'. Moreover, it seems that the physics educators preferred to compare concrete situations, while the mathematics educators to compare the mathematical models. In the third place we find the category of '*Relate the empirical (concrete) situation with the represented mathematical model*'. Finally, only a few educators responded with '*judging students' response with no further comments*'.

The *type of communication* suggested by the educators was mostly in the form of monologues (N= 16 out of 25 participants) and only N= 9 out of 25 proposed having a discussion with the students.

We provide characteristic responses and their analysis below:

R1 '*I would use the definition of the periodic function in order to prove that the graph represents a periodic motion* (Q20_math)

Response	*Type of reasoning*	*Type of communication*
R1	Use nomo-logical modes of reasoning (by applying the definition).	Monologue

R2 '*In the case of the swing we have frictions, so as the time goes on, the maximum and the minimum of the graph changes, as a result it is not a periodic motion.*' (Q2_math).

Response	*Type of reasoning*	*Type of communication*
R2	Relate the empirical (concrete) situation with the represented mathematical model	Monologue

R1 'There are many periodic motions that are not modeled by a sinusoidal graph, for example, the motion of the earth around the sun (Q28_physics)

Response	*Type of reasoning*	*Type of communication*
R1	Compare same type of representations (concrete situations)	Monologue

R2 'I would discuss with the students the relation of the phenomenon as it happens in nature with their mathematical models (Q15_physics)

Response	*Type of reasoning*	*Type of communication*
R2	Relate the empirical (concrete) situation with the represented mathematical model	Dialogue

R3 '*The answer is not right because* $f(x+T) \neq f(x)$(Q6_math)

Response	*Type of reasoning*	*Type of communication*
R3	Use nomo-logical modes of reasoning (by applying the definition).	Monologue

R3: '*I would present the sinusoidal curve and I would discuss with my students the similarities and the differences with the two curves (*Q8_ math)

Response	*Type of reasoning*	*Type of communication*
R3	Compare same type of representations (mathematical models)	Dialogue

R3 '*The graph does not represent a periodic motion (*Q26_math)

Response	*Type of reasoning*	*Type of communication*
R3	Judging students' response as correct or incorrect with no further comments.	Monologue

R2: *'It represents a decreasing oscillation, the amplitude decreases, I would provide the example of the pendulum clock and I could ask them to compare the two motions.'* (Q18_eng).

Response	*Type of reasoning*	*Type of communication*
R1	Compare same type of representations (concrete situations)	Dialogue

Questionnaire - Task 4

In this task we asked the educators to write down their suggestions on how they could help their students to develop a unified view on periodicity where aspects of the notion from the different subjects coexist harmonically.

Educators made many suggestions on this issue. The categories emerged in this task were

- provide many examples of everyday life periodic phenomena *(e.g. the periodic motion of the earth around the sun*)
- define & explain carefully the mathematical models
- use digital technology (e.g. animations of period motions)
- Help the students make links of the concrete situations with the mathematical objects that they model them.
- propose the use of inter disciplinary projects and co-operate with educators from other subjects

The quantitative analysis of the 26 educators who responded in this task we take the following results. We have to mention that some educators made more than one suggestion.

TABLE 4.9: Educators' suggestions	
Categories	*Frequency* *N=26*
Provide many examples of everyday life periodic phenomena *(e.g. the periodic motion of the earth around the sun*).	12
Define and explain carefully the mathematical models.	10
Use digital technology (e.g. animations of period motions).	7
Help the students make links of the concrete situations with the mathematical objects that they model them.	3
Propose the use of inter disciplinary projects and co-operate with educators from other subjects.	3

In first place was the suggestion: '*provide many examples of everyday life periodic phenomena*'. Educators from all subjects proposed this category. In second place was the suggestion on '*careful definition and explanations of the mathematical models*', and it was common for all subjects, too. The '*use of digital technology'* was mentioned by a few educators. The last categories of '*relating the concrete situation and their mathematical models*' and '*the use of inter disciplinary projects*" were proposed only by engineering and physics educators.

Activity 2- interviews

The interview theme in this research activity was based on the role of the argumentation developed in textbooks when topics on aspects of periodicity are presented in educators' classroom practices.

We focus our attention on different topics according to each participant: For mathematics teachers the topics are: Define periodic functions & Study the sinusoidal function (Mathematics, 2nd year in upper secondary school). For physics teachers: Define periodic motions (Physics, 3rd year in lower secondary school) and Define simple harmonic oscillation (Physics, 2nd year in upper secondary school). The teachers in engineering courses choose the topic themselves.

Interview - the role of everyday examples when introducing the notion of periodicity

We asked the educators about the role of every day examples when they introduce the notion of periodicity in their classrooms. Some questions were: *Do you use in your* *teaching practice the examples provided in the textbooks or other examples*? *Can you specify*? *How do you connect the examples with the topic you teach*? *Why do you use examples for?*

The analysis of the interviews of all participants gave us the following results:

When introducing the notion of periodicity all the educators use every day examples in their lesson in most cases not included in their textbooks. These examples could be "*the motion of the pendulum clock*" e.g. int10_ph or "*the motion of the playground swing"* (int11_math or *"the weekly publication of magazines"* (int2_eng) or natural phenomena taken such as the *"the day-night shift"* or "*the phenomenon of the tides"* (int4_math).

Fig. 4.3: A microcosmic example [int1_physics]

The use of digital technologies as teaching tools (videos and educational software e.g. 'interactive physics') is mentioned by six educators (int12_math, int1_physics, int5_eng, int2_physics, int6_physics & int10_physics). For example, one educator mentioned that sometimes in his class uses videos showing periodic macrocosmic and microcosmic phenomena such as *"the oscillation of the molecules in solids*" (int1_physics) (see Fig. 4.3). The participant added that the students were more interested in periodic macrocosmic phenomena taken from the subject of astronomy than microcosmic. He explained this as follows "*they do not have direct experiences with the microcosmic phenomena as opposing to macrocosmic ones (the celestial phenomena) through the NASA observations and their own experiences*" (int1_physics).

The use of the example of the menstrual cycle was mentioned by three educators (int1_physics, int5_eng & int9_eng). One of them made the following remark "*It was surprising for me that pupils, mostly the girls, mistook the word 'period' [a common phrase used for this phenomenon in Greek] as the 'time length of the bleeding phenomenon' and not the 28-day length of the menstrual cycle*" (int9_eng).

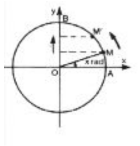
Two educators (int12_math & int10_physics) were critical of using graphical representations as models that exhibit a periodical behavior of every day situations in their teaching. *"I think that there is no real situation that is described by a particular mathematical model and this is what I want my students to understand"* (int12_math). Educators claim that their students could realize the above issue by practicing on specific tasks with interactive digital technologies software. An educator referred to the task "*the medium temperatures in every month for ten years in the city of Athens*". As the educator mentions, *"although the phenomenon could be modeled by a periodic function not all data are presented on the graph"* (int12_math).

In the question "*why do you use everyday examples of periodic phenomena for*?" all participants responded that students are interested in participating in such activities. Some of them added "*students at the end of the school year mostly remember those examples and not the theoretical discussions*" (int10_physics). Besides, all the engineering educators responded by emphasizing the circular behaviour of periodic phenomena e.g. "*these examples are helpful for the students to understand that many everyday phenomena are repeated in a circular way*".

It was interesting, though that in the question *"How do you connect the examples with the topic you teach*?" it seems that in most cases the examples used were disconnected from the thematic unit. Only one educator mentioned that he tried to use them as generic examples. Particularly, after asking his students to calculate the period of several periodic examples he wanted them to provide a definition of the word 'period of a periodic phenomenon'. To his surprise, this task was very hard for his students

> *"it seems that for students in this age [he refers to age 15, or 3rd grade in lower secondary school] it was almost impossible to generalize from particular situations or proceed from the particular and the concrete to the abstract and the general [...] when I gave them the definition they could not understand why their definition was incomplete*" int1_ph.

Interview - The utilization of the knowledge organization developed in school textbooks on specific topics.



[The school text] We notice that as x values from 0 to π/2, point M moves from A to B. Therefore, the y-coordinate increases, thus the function sinx is strictly increasing in the interval [0, π/2]. Similarly, we find that the function is strictly decreasing in the interval [π/2, π]. [...] Moreover, the function has a maximum value on x=π/2 (sinx=1) and a minimum value on x=3π/2 (sinx=-1).

Fig. 4.4: Example of an omitting mode of reasoning (int4_math).

Some questions were: *In the school text the new knowledge is developed in a certain way. Do you follow this when you teach one of above topics*? *Are there some parts on the development of the new knowledge that you pay more attention to when you teach this topic*? *If yes, which exactly*? *How is the conceptual understanding influenced by the above activities?*

In our study we consider that the new knowledge organization presented in a textbook is arranged in a specific way and presents a type of argumentation. This argumentation is considered as composed by a series of modes of reasoning (MsoR) that the author develops in a text when organizing and presenting the new knowledge.

Most of the participants express many concerns about the textbooks *e.g. "I think that the books say many things but not purposely ... they tell a story but nothing in particular"* (int6_physics). As a result, they prefer to modify parts of the knowledge organization provided in each thematic unit. The modifications mentioned by the educators could be omitting or adding or changing parts of the textual information. These modifications could result in omitting or adding modes of reasoning and hence changing the argumentation developed by the author.

The case of omitting parts of text is mentioned by two mathematics participants. The reason for this change was *'to gain time in order to give my students more information'* on issues considered by them as more important. Particularly, the participants mentioned that usually (a) instead of using two generic examples in order to define periodic functions they use only one example (mostly the example of the playground swing) (int8_math) and (b) they prefer not using the trigonometric circle in order to study the sinusoidal function.

I present the following extract of the discussion with the educator about the thematic unit: 'Study the sinx function', mathematics 2nd grade, Upper secondary school.

Researcher: Why do you omit using the trigonometric circle?

Educator: students do not realize easily that the sinusoidal function has period 2π. They ask me 'what is π?' so I start reminding them how we defined 'π' in geometry. So, after omitting the part with the trigonometric circle I could go straight to the value table that they know and I use that to sketch the graph. Besides I have to explain to them that the sine function is odd and recall issues as oddness and evenness of functions ...when I start this chapter [trigonometry] I am always very anxious because I feel that I will not have the time to explain so many things (int4_math).

In this case, the educator changes the argumentation presented in the school text because he values some parts of it as difficult for students, and at the same time he adds other modes of reasoning that he thinks are more important for his students (e.g., defining 'π', a nomo-logical mode of reasoning that is absent in the text).

In the following two extracts educators mention why they usually omit a particular example when teaching the thematic unit 'Periodic motions', 3rd grade lower secondary school (Fig. 4.5).

The school text: [...] The muscle of the heart performs a periodic motion as well, as represented in the electrocardiogram [photo 4.2]

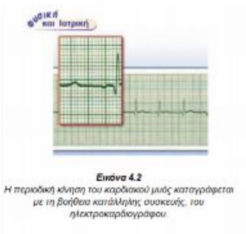


Fig. 4.5: An example of an omitted example

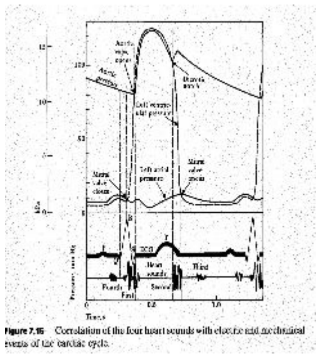


Fig. 4.6: The epistemological background of the cardiac cycle

Educator1: I never use the electrocardiogram as an example of a periodic motion, its is a very strange example, since the conditions should be perfect in order to have such a diagram [int6_physics]

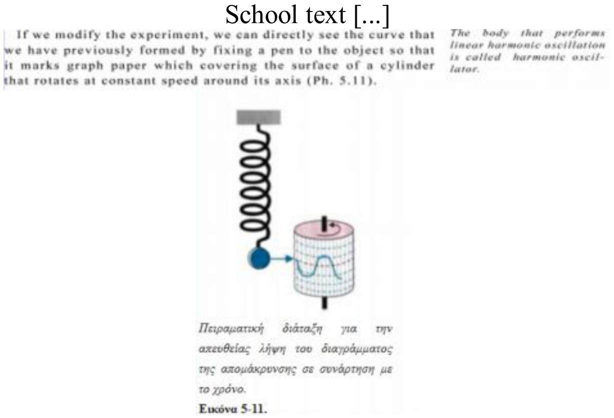
Educator2: I prefer not to use the example with the electrocardiogram since it is hard for me to explain what parts of this graph are represented [int3_physics].

The reason for this change in the knowledge organization developed in the textbook is either because they consider it is not representing a real situation or because the context is unfamiliar to them. In Fig. 4.6 we present the analysis of the graphical representation as presented in a medical book. In this way we understand the difficulty of the context of this particular VR and the educators' reluctance to use it in their class.

Two physics participants mentioned that instead of proving the formula T=2πl/g (the proof is in 3rd grade upper secondary school physics textbook), they prefer to provide the formula and use the interactive physics software in order to *verify* that the period of the oscillation is related to the above quantities in the way represented in the formula. In this way they change a mathematical mode of reasoning to a logical-empirical one. They mention that in the verification process the students were more involved than in the proof practice and they are "*more convinced of its truth*". As an educator mentions "*For many years the students thought that the formula is something coming out of the blue, now the digital technology could help students to realize that this is not the case"*. According to the educator this is very important since "*it influences students' understanding*" (int3_physics).

In the following extract the educator changes the whole argumentation proposed by the textbook in the thematic unit 'Define simple harmonic oscillation' 2nd grade in upper secondary school. Particularly, instead of using two logical-empirical modes of reasoning in order to define the linear harmonic oscillation (one specific-general and one general-specific) he prefers using a logical-empirical and an empirical one. The logical-empirical is the reasoning that involves a video presentation (see Fig. 4.7 (b)) while the empirical one is students' enactive participation in producing the sinusoidal curve. In the following extract the participant explains his choice as follows:

School text [...]



Educators' alternative practice

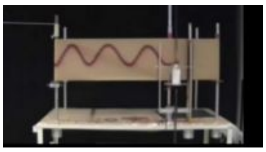


Fig. 4.7: (a) The mode of reasoning met in school text; (b) the alternative approach suggested by the educator.

Educator: I never use all the examples proposed by the text. Instead I prefer using the following video [...] In this video the students see why the displacement of a body that oscillates on an ideal spring is described by the sinusoidal curve.

Researcher: Why not using a part of the thematic unit where an experimental set up directly graphs the sinusoidal curve as well?

Educator: I think that this experiment is impossible to be successful. Besides, my students can carry out this experiment by themselves, ... one student holds a pen in his hand and moves his hand vertically while his colleague moves a paper before him with a constant speed. The pen sketches the sinusoidal curve on the paper [int1_physics].

Adding modes of reasoning seems to be a common practice in all educators. The mathematics teachers mostly add definitions of notions that they consider important in their teaching practice (e.g., defining the notion of 'π'). These definitions are parts of previous thematic units or other mathematical subjects. On the other side, physics educators add parts of argumentation met in mathematics texts. For example, in int3_physics the educator mentions: *"when I teach the linear harmonic oscillation I have to define the sinusoidal function, for this reason I sketch the trigonometric circle* *and take certain values, T, T/4. T/2 etc. ... this is the only way for the students to follow my lesson".*

An engineering educator changes a sinusoidal image as follows when he teaches "Alternate currents" to his students:

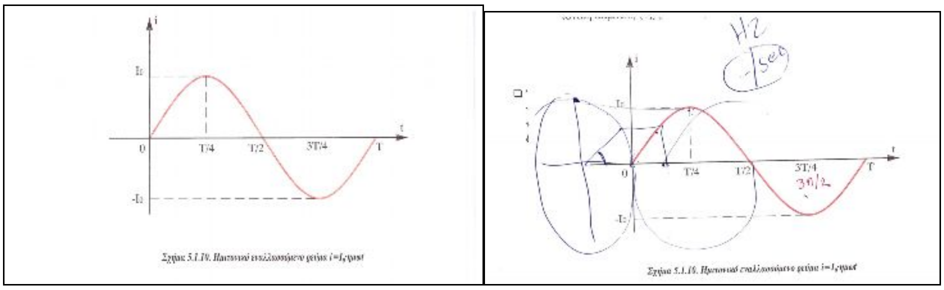


Fig. 4.8: the textbook VR and the way the educator changes it in order to make the notion circle per second or Hertz clear:

> "*We have the sinusoidal curve which shows [the values] 0 - max - 0 - min and I tell them if this part of the curve [the second part] I flip it under the first part it makes a cycle ... that is why we refer to the cycle per second which is the Hertz ... and when I ask them how many times in a second it makes a circle ... then I reflect this circle on the y- axis and I have another circle this is the trigonometric circle, the difference is that in my first circle I move clockwise on the second circle ... I move counter clockwise I project the maximum value of the sinusoidal curve to the trigonometric circle and I say ... this is why the sine of 90 degrees is 1 ... this is useful for a later topic afterwards*" [int5_eng].

Finally, changing the illuminating role of a mode of reasoning (in the physics textbook usually after making a general claim some examples are added) to an investigating one seems to be also a common practice for some educators. For example, in int6 the educator mentions: "*sometimes after presenting the definition of a periodic motion to my students I ask them to give me some examples, sometimes their examples are not qualified as appropriate, I ask them is it a motion? Do you think that it is periodic?*" (int6_physics).

Ch. 5 Concluding remarks

Periodic notion is recognized by the educators in a wide range of teaching units, hence teaching aspects of the notion of periodicity constitute an important part of everyday students' school practice.

The use of images of periodicity in educators' teaching practices

According to our data, three are the most fundamental images of the notion: The sinusoidal function, the trigonometric circle and the playground swing.

The sinusoidal function

The sinusoidal curve is qualified as the prototypical image of the notion across subjects while the sinusoidal function seems to be a common teaching tool for engineering, mathematics and physics educators. Mathematics educators mention using sinusoidal functions either to solve trigonometric equations or to study their characteristics (e.g., period, maximum values, etc.) while physics and engineering educators mention using this function to model the periodical change of a number of quantities in the course of time (Act 1/ task 1).

The trigonometric circle

The image of the trigonometric circle is a common teaching tool in the three subjects. Although it was mentioned only by mathematic educators in Act1/task 1, VR2 that contains this circle is chosen by many educators (VR2 took the second place in educators' potential and actual use) while many educators in Act2 mention using this in their classroom practice.

This circle is used in teaching practice to graph the sinusoidal function. So, the above two images are related in a functional way.

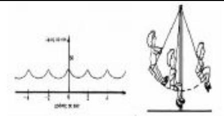
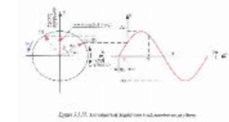
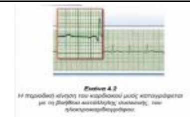
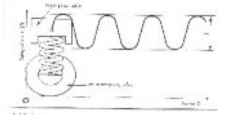
The playground swing

The playground swing seems to be a common image of periodicity in mathematics and physics. We argue on that because in act 1/task 1 this image is mentioned by a number of participants who teach mathematics and physics while the same image is chosen mostly in act1/task2. This image is used as an example of an oscillating object in physics texts in its abstract form as a simple pendulum or in its concrete form as a pendulum clock. Both images are mentioned by the participants. Despite the above, some educators consider as inappropriate for their students to use this example when justifying that a graph represents a periodic motion (Act1/task 3).

The above three images exhibit different levels of generality (the sinusoidal function and the trigonometric circle are abstract images while the playground swing illustrates a concrete situation). The students meet the above images in almost all subjects as they are supposed to synthesize them in order to gain an understanding of periodicity.

In Act1/task2 we selected five VRs (and the accompanying captions) of the notion of periodicity, as we found them in five different school texts (two from physics texts, one from a mathematics text, and two from engineering texts). All VRs represent images of periodic motions in different ways. We include sinusoidal graphs (VR2, VR4 & VR5) and non-sinusoidal graphs (VR1 & VR3). The genres of each representation are different (VR1 & VR4 are photos, VR3 & VR5 are a combination of drawings and graphical representations and VR2 is a combination of a schematic representation (the trigonometric circle) and a sinusoidal graph. The context of graphs is different as well. VR1 represents a part of an electrocardiogram, VR3 represents a playground swing, VR4 represents the elaborated motion of an ideal spring and VR5 the vibration of a car due to its suspension system. Finally, VR2 is a context-free image. VR3, VR2, VR4 and VR1 were selected by many participants with the following percentages 78%, 66%, 56% and 46% respectively. Only a few educators (22%) selected VR5 for their teaching (the educators could select more than one VR either for their actual/current or for their potential use in class).

TABLE 5.1: The five images of periodicity (VRs) according to the educators' selection

 VR3	 VR2	 VR4	 VR1	 VR5

It seems that the level of generality is not always the educators' criterion for using an image in their teaching practice but the content and the context of the image. Particularly, their preference seems to depend on (a) how central they consider it in their teaching and (b) how they could handle it in the class. It is interesting that many mathematics educators suggested using VR4 (an elaborated photo image) in their class although in Greek mathematics texts the presence of photos is almost rare. Also, it is interesting that although in physics texts the only periodic graph is the sinusoidal curve they value other types of curves that exhibit a periodic behaviour not present in their text as important in their teaching as well. The electrocardiogram was not a familiar image to them although it was met in 3rd grade in lower secondary school. In the interview activity the educators express their concerns about this particular image.

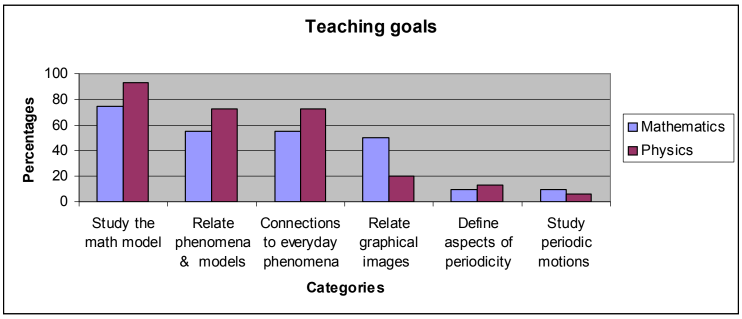
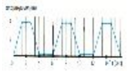
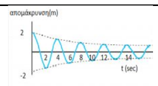


Fig. 5.1: Mathematics and physics educators' teaching goals.

The teaching goals mentioned by mathematics and physics educators are presented in Fig. 5.1. Mathematics and physics educators appear to have almost the same teaching goals when using the above VRs. Particularly, modeling is considered by them as their prior teaching goal (either by studying the mathematical model or by relating the phenomena to their mathematical models). This is in cohesion with participants' responses in Act 1/task4 when they value the case of "Define and explain carefully the mathematical models" high in order to help students develop an understanding of the notion. Moreover, 'make connections to every day phenomena' is valued high even for mathematics educators and this result indicates that educators struggle to find images that could help them to make connections to every day life. Although in textbooks some of the above VRs were used as fundamental images in order to define notions, defining certain aspects of periodicity was not acknowledged by the educators as a teaching goal. For example, VR3 in mathematics textbook was used as a generic example in order to define periodic functions, but only in two cases was it acknowledged in this way by the educators.

If we examine the teaching goal for each VR separately, we see that each educator have different learning objectives even in the same subject. Since different learning objectives could produce different teaching practices, we conclude that the use of the same image in school practice does not necessarily presuppose the same activity.

Educators' modes of reasoning when handling students' responses

Students' Task		
Graph 1	*Graph 2*	*Questions*
		*Does this graph represent a periodic motion? Justify your answer.*
The students' responses		
R1: Graph 1: 'It is not periodic because the function (represented) is not sinusoidal'		
R2: Graph 2: 'It is periodic because it represents the motion of the swing.'		
R3: Graph 2: 'It is periodic because every sinusoidal function is periodic.'		

Most educators suggested the use of definitions (we identify them as nomo-logical modes of reasoning) in order to handle students' responses in class either R1 or R2 or R3. There is a small advantage for mathematics educators in this category though (see Fig. 5.2). In the second place we find the categories that had to do with comparisons either "same type of representations" (compare 1) or "the empirical (concrete) situation with the represented mathematical model" (compare 2) (mostly in the case of R2) (Fig. 5.2). This comes in conflict with the students' responses (2nd research phase) where this type of reasoning was almost rare.

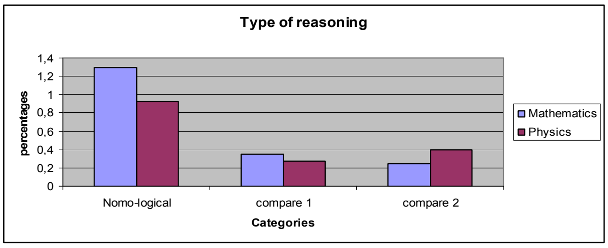


Fig. 5.2: Educators' type of reasoning when handling students' responses.

The use of everyday examples when introducing the notion of periodicity

All the educators mention a lot of examples used in their lesson when teaching aspects of periodicity. These examples vary from phenomena close to students' experiences (menstrual cycle) to natural phenomena taken from the microcosm (the vibrations of the particles) and the macrocosm (the motion of the planets) (Act 2/theme1). Besides, many educators suggest that making connections to everyday life periodic phenomena could hep students to develop a unified view of periodicity (Act 1/task 4).

From the analysis of our data in Act 2/theme1 it seems that the educators is not consciously use every day phenomena as generic examples or in order to make a general claim but only to stimulate their students' attention. This educators' attitude is in conflict with the use of examples in textbooks. Examples in textbooks were used purposely either to make generalizations (generic examples) or to provide applications of the notion presented in the thematic unit. "*Is one generic example enough to make a general claim*" is another issue that came up from our data. Some educators mention omitting the use of all the examples provided in the texts and they think that one example is enough.

The utilization of new knowledge organization developed in school textbooks on specific topics.

In general, all the educators were critical of the textbooks use in their classroom practice. Consequently, they prefer to modify parts of the new knowledge organization provided in each thematic unit. The modifications mentioned by the educators could be omitting or enriching the modes of reasoning presented in their texts. These modifications could result in changes in the argumentation developed in each thematic unit. The case of omitting a part or the entire mode of reasoning is suggested by some educators due either to lack of time (e.g., not reasoning by using the trigonometric circle in mathematics) or valuing it as inappropriate (e.g., the electrocardiogram example in physics) or unnecessary (providing one generic example instead of two in mathematics). On the other side, three educators (two physics and one engineering) mention that they usually include the reasoning by using the trigonometric circle in their class even in the case this reasoning does not appear in their texts since they value it as important in order to define the sinusoidal function. In this case a mode of reasoning disappears in mathematics educators' practice and appears in physics' educators practice. The case of enriching the modes of reasoning presented in their texts could be either by adding definitions (expressed by two mathematics educators) or adding logical-empirical and empirical modes of reasoning (expressed mostly by the physics educators). All the above modifications are made for enhancing their students' learning objectives.

We conclude that the above modifications could result in changing the argumentation developed and hence influence students' conceptualization.

Co-operation among educators

Co-operation of educators from different subjects seems to be a non-preferable suggestion by almost all educators. This indicates an attitude towards non co-collaboration among educators (Act1/task4 & Act2/theme 3). The educators never had the chance to see textbooks of other subjects (Act2/theme 3). This case involves both mathematics and physics educators. Physics educators never had the chance to see how mathematics textbooks introduce the students to the notion or periodic function, nor how they address the sinusoidal curve. The same applies to the mathematics educators. The above outcomes come in conflict with the result in Act 1/task 2 where many educators mention the potential use of images taken from other texts.

Connecting mathematics, physics and technology instruction (two subjects that have many connections) is considered as a central issue in the contemporary research literature (e.g., Frykhlom & Glasson, 2005) since it can strengthen students' understanding of common notions as in our case the notion of periodicity.

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APPENDIX I
THE QUESTIONNAIRE

Tools and practices when teaching the notion of periodicity: the role of visual representations

Introduction

The current research is part of a research project within the framework of the action “Supporting Postdoctoral Researchers” of the operational program “Education and Lifelong Learning”. The study takes place in the School of Pedagogical and Technological Education (ASPETE).

Particularly, the current research examines the teaching and understanding aspects of the notion of periodicity. This questionnaire is for educators that teach mathematics, science, and/or engineering courses in lower, upper secondary and/or vocational schools whose courses include topics related to periodicity (e.g., periodic functions, periodic motions etc.).

We are looking for commonalities and differences in the teaching practices of educators in different subjects. Particularly, we explore the role of visual representations of the notion (e.g., photos, schematic representations, graphs) in teaching practice.

Your answers will be completely anonymous. Your responses are voluntary and will be kept confidential.

The supervisor	The researcher
V. Spiliotopoulou Prof. in Department of Education, ASPETE	Chrissavgi Triantafillou Post-doctoral reasearher, Department of education, ASPETE
Science education	Mathematics education email: chrtriantafillou@gmail.com

QUESTIONNAIRE

(A) PERSONAL DATA

DISCIPLINE:

TEACHING EXPERIENCE (in years):

Check **(√)** in which of the following educational levels you have taught.

Secondary school	Upper secondary school	Vocational upper secondary school

(B) The notion of periodicity- visual representations (e.g., photos, schematic representations, graphs) of the notion in school texts

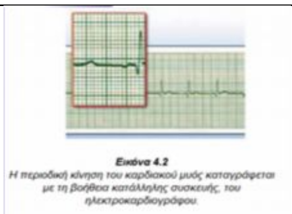
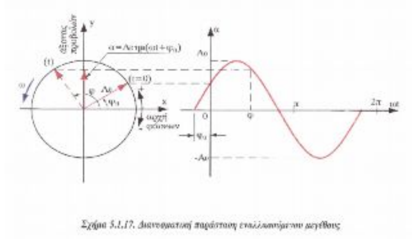
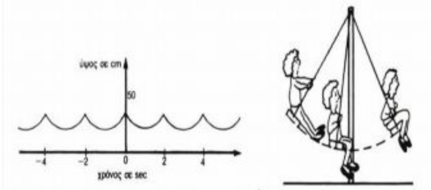
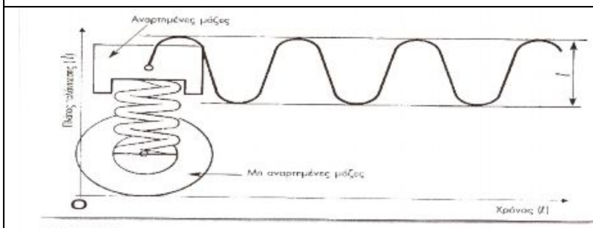
1) Please name the basic teaching units included in your courses in which the notion of periodicity is included.

2) Please chose one of the teaching units you named above and list the basic visual representations of the notion included in the textbook used for that unit.

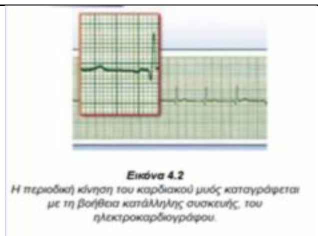
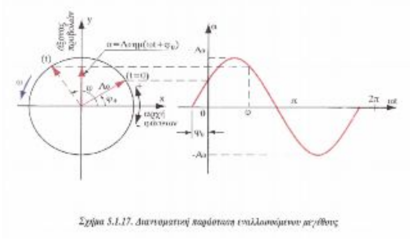
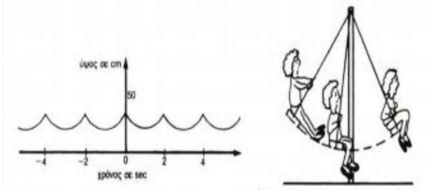
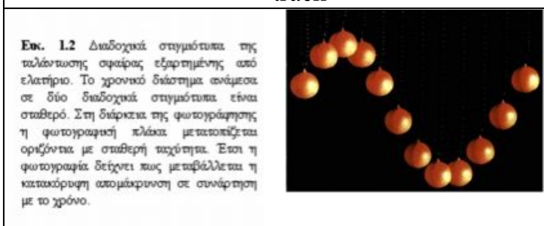
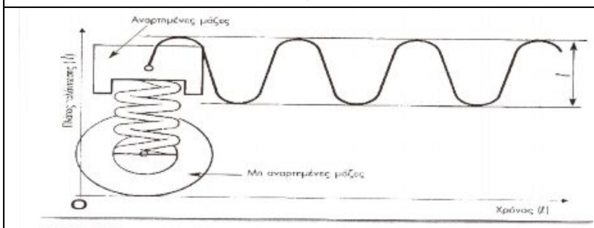
3) Now please chose only one of the visual representations you listed in question 2 above and describe how it is used (its role) in the specific teaching unit.

4) We provide below five visual representations (VRs) of the notion of periodicity taken from school texts.

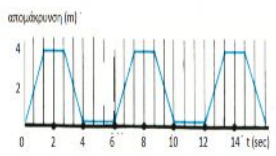
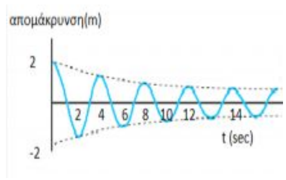
α) Which of them do you usually use in your teaching practice?

Visual representations (VRs)	Can you please describe your teaching goal or where you place emphasis?
 Physics, 3rd year at lower secondary school	
 Electrotechnology, 1st year, 1st circle, technical and vocational school	
 Algebra, 2nd year, upper secondary school, general track	
 Physics, 3rd year upper secondary school, scientific track	
 Fig. 4.29: Oscillation of a spring and the car without using an absorber Car systems, I, 2nd year in 1st circle, technical & vocational school	

b) For the VRs that you do not use in your class, which ones could you imagine implementing in your teaching? (Here, please do not consider the source textbook, but merely the task and its VR).

Visual representations (VRs)	Can you please describe your teaching goal or where you place emphasis?
 Physics, 3rd year at lower secondary school	
 Electrotechnology, 1st year, 1st circle, technical and vocational school	
 Algebra, 2nd year, upper secondary school, general track	
 Physics, 3rd year upper secondary school, scientific track	
 Fig. 4.29: Oscillation of a spring and the car without using an absorber Car systems, I, 2nd year in 1st circle, technical & vocational school	

5) The following task was given to undergraduate students at university and technological institutions. The task is based on interpreting graphical representations of VRs in school textbooks so it was possible to be used and in secondary school practices.

<table>
<tr><td colspan="3">Task</td></tr>
<tr><td>Graph 1</td><td>Graph 2</td><td>Questions</td></tr>
<tr><td>
</td><td>
</td><td>Does this graph represent a periodic motion? Justify your answer.</td></tr>
</table>

We provide below three characteristic student responses.

Responses	Briefly describe how you could handle these answers if they came up in your school class.
Graph 1: *"It is not periodic because the function (represented) is not sinusoidal."*	
Graph 2: *"It is periodic because it represents the motion of the swing."*	
Graph 2: "It is periodic because every sinusoidal function is periodic"	

6) What suggestions do you have that could help students develop a unified view on periodicity where aspects of the notion from the different subjects coexist in a harmonic way?

Thank you very much for your participation!

APPENDIX II

Two thematic units and their analysis (Act. 2)

Graphing the function sinx

Nomological, initial claim MsoR

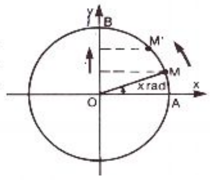
Since the function f(x) = sinx is periodic with period 2π, it is sufficient to be studied in an interval that has length 2π, e.g. [0, 2π]. As we already know, sinx represents the y-coordinate of the point M(x,y) on the unit circle. Therefore, it is sufficient to examine how y-coordinate of point M varies, when it rotates around the trigonometric circle towards the positive direction, starting from A.

Logical-empirical, specific-general, mathematical evidence MoR

We notice that:

- As x varies from 0 to π/2, the point M moves from A to B. Therefore, the y-coordinate increases, thus the function sinx is strictly increasing in the interval [0, π/2].

Similarly, we find that the function sinx:



— strictly decreases in the interval [π/2, π],

— strictly decreases in [π, 3π/2],

and strictly increases in [3π/2, 2π].

- The function has
- a maximum value on x = π/2, sinx = 1 and
- a minimum value on x = 3π/2, sinx =-1.

The results are summarized in the following table:

x	0		$\frac{\pi}{2}$		π		$\frac{3\pi}{2}$		2π
ημx	0	↗	1	↘	0	↘	−1	↗	0
			μεγ.				ελαχ.		

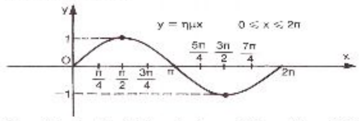
Mathematical, techniques MoR

In order to sketch the graph of this function, we need a table value. According to what we already know, we complete the next table.

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
ημx	0	$\sqrt{2}/2 \simeq 0{,}71$	1	0,71	0	−0,71	−1	−0,71	0

We represent the couples (x, sinx) with points on the Cartesian plane and we join them with a continuous line.

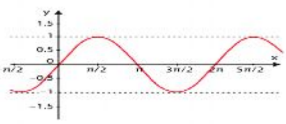
In this way, the following graph of the sinx function on the interval [0, 2π] is produced.



Since the function f(x) = sinx is periodic, with period 2π, the curve has the same shape in the intervals [2π, 4π], [4π, 6π] etc. and in the intervals [-2π, 0], [-4π, -2π] etc.

Nomological, Main claim MoR

This is depicted in the following graph of the **sinusoidal curve.**



Logical-empirical, explanatory MoR

Finally, we know that opposite angles have opposite sines. Therefore, for every x ∈ ℝ, we have sin(-x) =-sinx. This means that the function f(x)=sinx is odd and hence its graph has a point symmetry on O(0,0).

(Andreadakis, S., Katsargyris, B., Papastavridis, S., Polyzos, J., & Sverkos, A. (2012). Algebra, 2nd Year of Upper Secondary School, Common Core Subject, pp. 75-77. Athens, Greece: OEDB)

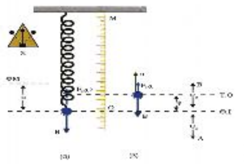
The analysis of the mathematics text

(5.2.) Linear harmonic oscillation with an ideal spring.

Empirical, systematic description MoR

a. Definitions - Basic magnitudes

In order to study the oscillation that a body performs with the help of a spring we need an ideal spring (of constant k and length L_0) a solid spherical body (mass m) a timer X and a tape measure M.



It is more correct to say that oscillation is performed not by the body, but by the system spring-body.

Photo 5.7

With the help of a timer, we find the T period of the oscillation counting the time for the route AOBOA or the route OBOAO or for every "cycle" and we notice that it remains constant.

We can also measure the time intervals for the routes AO, OB, BO and OA and notice that they are equal between themselves (so each one is equal with T/4).

Mathematical, techniques MoR

ψ	t
0	0
ψ_0	T/4
0	T/2
$-\psi_0$	3T/4
0	T

Photo 5.8: *Table with the values of the displacement on specific time instances*

With the help of all the above measurements we can complete a table (Ph. 5.8) of the displacement y in relation to the time of the movement t (for reason of simplicity we consider 0 the time when the body passes from the equilibrium position) and sketch the graph with the help of the curve y=f(t) (Ph. 5.9). But the table and the graph do not provide enough information.

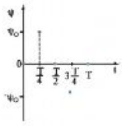


Photo 5.9.

Logical-empirical, specific-general, experimental evidence MoR

If we want to obtain more information, we can use (if possible) chronophotography, in which the object's motion has been photographed several times in different positions during one period.

Omitted text

The value chart y-t is thus quite thorough, so as for the y=f(t) curve, which we design with its help (Ph. 5.10), to be possible to be drawn continuous and to be considered very close to the real one.

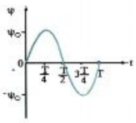


Photo 5.10: *The displacement is a sinusoidal time function*

Nomolological, main claim MoR

This curve has a sinusoidal form which is the characteristic feature of the linear harmonic oscillation.

Linear harmonic oscillation is called the oscillation that an object performs when its orbit is on a straight line and its displacement is a sinusoidal function of time.

(the sinusoidal curve is also called harmonic).

The body that performs linear harmonic oscillation is called harmonic oscillator.

Logical-empirical, explanatory MoR

If we modify the experiment, we can directly see the curve that we have previously formed by fixing a pen to the object so that it marks graph paper which covering the surface of a cylinder that rotates at constant speed around its axis (Ph. 5.11).

We can also measure with the help of this curve the displacement at different instants and draw the table of the values y-t.

Empirical, systematic description MoR

(It is obvious that in order for the experiment to succeed, the cylinder's rotation period should be bigger than the period f the oscillating body and the amplitude of the oscillation must be smaller than half of the height of the cylinder).

Omitted text

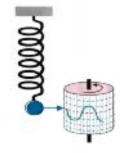


Photo 5.11: *Experimental set up for the direct sketching of the graphical representation of the displacement versus time*

(Alexaki, N., Ampatzi, S., Gkougkousi, I., Kountouri, B., Mosxovete, N., Ovadia, S., Peptroxeilo, K., Samprako, M., & Psalida, A. (2012). Physics, 2nd Year of Upper Secondary school, Common Core Subject, pp.204-206. Athens Greece: OAED)

The analysis of the physics text